Gutenberg School of Management and Economics

& Research Unit “Interdisciplinary Public Policy”

Discussion Paper Series

On the One-to-One Pickup-and-Delivery Problem with Time Windows and Trailers

Michael Drexl

October 04, 2018

Discussion paper number 1816

Johannes Gutenberg University Mainz
Gutenberg School of Management and Economics
Jakob-Welder-Weg 9
55128 Mainz
Germany
https://wiwi.uni-mainz.de/

Contact details

Michael Drexl
Faculty of Applied Natural Sciences and Industrial Engineering
Deggendorf Institute of Technology
Deggendorf
Germany
and Chair of Logistics Management
Gutenberg School of Management and Economics
Johannes Gutenberg University,
GermanyJakob-Welder-Weg 9
55128 Mainz
Germany

Michael.drexl@th-deg.de

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Technical Report LM-2018-05

Michael Drexl
Faculty of Applied Natural Sciences and Industrial Engineering,
Deggendorf Institute of Technology, Deggendorf, Germany
and
Chair of Logistics Management, Gutenberg School of Management and Economics,
Johannes Gutenberg University, Mainz, Germany
E-mail: michael.drexl@th-deg.de

4th October 2018

Abstract

This paper studies an extension of the well-known one-to-one pickup-and-delivery problem with time windows. In the latter problem, requests to transport goods from pickup to delivery locations must be fulfilled by a set of vehicles with limited capacity subject to time window constraints at locations. The goods are not interchangeable; what is picked up at one particular location must be delivered to one particular other location. The extension discussed here consists in the consideration of a heterogeneous vehicle fleet comprising lorries with detachable trailers. Trailers are advantageous as they increase the overall vehicle capacity. However, some locations may be accessible only by a lorry without a trailer. Therefore, special locations are available where trailers can be parked while lorries visit accessibility-constrained locations. This induces a nontrivial tradeoff between an enlarged vehicle capacity and the necessity of scheduling detours for parking and reattaching a trailer.

The contribution of the present paper is threefold: (i) It studies a practically relevant generalization of the one-to-one pickup-and-delivery problem with time windows. (ii) It develops an exact amortized constant-time procedure for testing the feasibility of an insertion of a transport task into a given route with regard to time windows and lorry and trailer capacities and embeds this test in an adaptive large neighbourhood search algorithm for the heuristic solution of the problem. (iii) It provides a comprehensive set of benchmark instances on which the running time of the constant-time test is compared with a naïve one that requires linear time. The results of computational experiments show significant speedups of one order of magnitude on average.

Keywords: Vehicle routing; Pickup-and-delivery; Trailers; Adaptive large neighbourhood search; Insertion heuristic; Constant-time feasibility test.

1 Introduction

The one-to-one pickup-and-delivery problem with time windows and trailers (PDPTWT) can be described as follows. There is a set of requests or tasks to transport specified amounts of goods between paired pickup and delivery locations. To fulfil the tasks, a set of capacitated vehicles consisting of *single lorries* and *lorry-trailer combinations (LTCs)* is available. Each vehicle has a given start and a given end location. The start location of a vehicle may differ from the vehicle's end location. A trailer has the same start and the same end location as its associated lorry. Each single lorry and each LTC has a fixed cost, incurred only if it fulfils at least one task, and a travel cost for moving from one location to another. Fixed and travel costs may differ between vehicles; for LTCs, travelling between two locations with the trailer attached may be more expensive than without. Capacities may also differ between vehicles. LTCs have a lorry capacity and a trailer

capacity. After picking up and before delivering the goods of a certain task, vehicles may visit other pickup and/or delivery locations. All pickup and all delivery locations can be visited by a single lorry and by an LTC lorry without its trailer. However, some pickup and some delivery locations may have accessibility constraints in the sense that they cannot be visited by an LTC lorry when the trailer is coupled. Because of these accessibility restrictions, there are also *parking and transshipment locations (PTLs)*. At PTLs, trailers can be decoupled, parked, and re-coupled, and load can be transshipped between an LTC lorry and its trailer. In this paper, a fixed lorry-trailer assignment is assumed. This means that each trailer can be pulled only by one lorry, and only this lorry can transfer load to or from the trailer. All task locations, i.e., all pickup or delivery locations, can be visited by any lorry, all locations designated as reachable by trailer can be visited by any trailer, and PTLs can be visited by all LTC lorries and trailers. Each task location is visited exactly once, whereas PTLs can be visited more than once by the same or different LTCs. The load to be picked up at a task location can be split arbitrarily between a lorry and its trailer if the location is visited by an LTC.

Each location has a single, hard time window that may be equal to the length of the planning horizon and thus nonrestrictive. Arrival at a location before the start of its time window is allowed and incurs waiting time but no cost. Waiting time is not limited. There are fixed service times at all task locations and all PTLs. At PTLs, there are two service times, one for the decoupling and one for the re-coupling operation. Travel times between locations and service times are independent of the current vehicle, of its current load and, for LTCs, of whether or not the trailer is attached. Travel and service times as well as fixed and travel costs are time-independent. All vehicles are available throughout the complete planning horizon.

An LTC route may visit any location and is partitioned into the *main route*, which is the part of the route where the lorry pulls its trailer, and zero or more *subroutes* that start and end at a PTL where the lorry parks its trailer while visiting one or more task locations. An LTC lorry may perform several consecutive subroutes starting and ending at the same PTL before finally pulling away its trailer. If a delivery location is visited on a subroute and the corresponding pickup location has been visited before this subroute, it must be ensured that the entire amount of goods bound for this delivery location is on the lorry at the start of the subroute. This may require a load transfer from a trailer to its lorry at a PTL.

There is no congestion at PTLs: arbitrarily many trailers can be parked at a PTL at the same time. Without loss of generality, it is assumed that a load transfer, if any, between an LTC lorry and its trailer takes place only directly before a decoupling operation, not when re-coupling. The duration (service time) of a decoupling operation includes time for a potential load transfer.

The problem is static and deterministic, i.e., all data are known in advance.

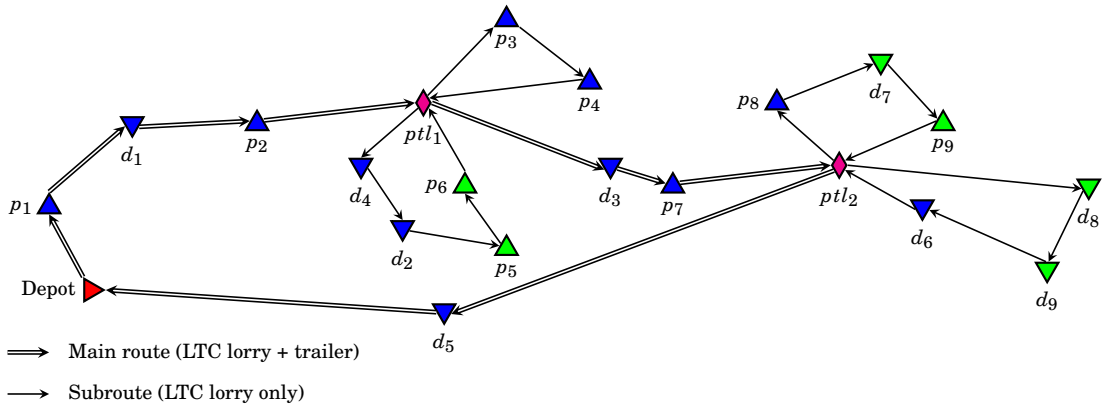
The objective of the PDPTWT is to find a feasible solution with a minimal (or, at least, low) sum of fixed and travel costs. A feasible solution consists of a set of feasible routes, one for each single lorry and one for each LTC, so that each task is covered by exactly one vehicle (single lorry or LTC). A route is feasible if and only if it starts at the start depot of the vehicle that performs the route, fulfils zero or more tasks, and ends at the vehicle's end depot, while maintaining all time windows, accessibility constraints, and lorry and trailer capacities. In a feasible solution, the following nine cases are possible with regard to accessibility constraints:

		Pickup can be visited with a trailer					
		yes	yes	no	no		
Delivery can be visited with a trailer	yes	1	2			yes	Pickup is visited on main route
	yes	3	4	5	6	no	
	no		7			yes	
	no		8		9	no	
		yes	no	yes	no		
		Delivery is visited on main route					

Figure 1 shows an example LTC route that fulfils the nine tasks $t_1, \ldots, t_9$. For $i = 1, \ldots, 9$, p_i and d_i respectively denote the pickup and the delivery location of task t_i. The route starts and ends

at the depot bottom left and performs four subroutes, two each at the parking and transshipment locations ptl_1 and ptl_2. In the figure, task t_i corresponds to case i of the above table for $i = 1,\ldots,9$.

Figure 1: Example LTC route



There is no lack of practical applications of the PDPTWT. This author has seen use cases in the supply of supermarkets, beverage stores, and apparel stores, in the transport of ready-mixed concrete garages and commercial waste bins, and, most notably, in the less-than-truckload business, where containers, swap-body platforms, and smaller collective consignments are transported by LTCs.

The contribution of this paper is threefold: (i) It studies a practically relevant extension of the one-to-one pickup-and-delivery problem with time windows. Put differently, it generalizes vehicle routing problems (VRPs, i.e., problems where either all pickups or all deliveries take place at a central depot) with trailers to pickup-and-delivery problems. (ii) It develops an exact amortized constant-time procedure for testing the feasibility of an insertion of a task into a given PDPTWT route concerning time windows and lorry and trailer capacities and embeds this test in an adaptive large neighbourhood search algorithm for the heuristic solution of the problem. 'Exact' means that the testing procedure will declare the insertion as feasible if and only if the route resulting from the insertion is feasible. 'Amortized constant-time' means that the test itself takes constant time and is independent of the number of tasks (or, equivalently, the number of locations visited) on the route, but that the test uses auxiliary data which must be computed in a preprocessing step which does not run in constant time. (iii) The paper provides a comprehensive set of new benchmark instances and empirically compares the running time of the constant-time test on these instances with a naïve one that requires linear time.

The rest of the paper is structured as follows. The next section gives a brief review of related literature. Section 3 presents the adaptive large neighbourhood search procedure used to solve the PDPTWT. In Section 4, the insertion feasibility tests regarding time and capacity are described. Section 5 presents the newly created benchmark instances and discusses the computational results obtained on them. Finally, Section 6 gives a conclusion and proposes topics for further research.

2 Related Work

This section briefly reviews pertinent literature, focussing on works concerned with pickup-and-delivery problems, routing problems with trailers, and efficient feasibility tests in heuristics for routing problems.

Pickup-and-delivery problems (without trailers) exist in several variants (one-to-one, one-to-many-to-one, many-to-many, simultaneous delivery and pickup) and have been extensively studied in the last decades. Important surveys are presented by Parragh et al. [26, 27], Doerner and Salazar-González [9], and Battarra et al. [2]. These works also provide classification schemes for the dif-

ferent variants. The static, deterministic, multi-vehicle, one-to-one variant with time windows is the most widely studied type. Exact algorithms for this problem are presented by Ropke et al. [34], Ropke and Cordeau [33], and Baldacci et al. [1]. According to Battarra et al. [2], the most successful heuristic procedures, by Bent and Van Hentenryck [3] and Ropke and Pisinger [35], are both based on large neighbourhood search.
Routing problems with trailers have also attracted a lot of interest from researchers. The surveys by Prodhon and Prins [32] and Cuda et al. [7] contain sections on VRPs with trailers, which are commonly referred to as truck-and-trailer routing problems (TTRPs). Most works on TTRPs consider no time windows. Exact algorithms for TTRPs with time windows (TTRPTWs) are presented by Parragh and Cordeau [25] and Rothenbächer et al. [36]. Heuristics for TTRPTWs are described by Drexl [10] (heuristic column generation), Lin et al. [22] (simulated annealing), Derigs et al. [8] (hybrid local and large neighbourhood search, attribute-based hill climber), and Parragh and Cordeau [25] (adaptive large neighbourhood search).
Pickup-and-delivery problems with time windows and trailers are less well studied. Most papers on this topic consider approaches for problems where vehicles consisting of a tractor and a semi-trailer are employed to perform full-load tasks, i.e., where a vehicle can transport only one task at a time. Examples are the problems examined by Cheung et al. [6] (attribute-decision model), Xue et al. [46] (tabu search) and Tilk et al. [42] (branch-and-price-and-cut). Concerning the PDPTWT version studied here, this author is aware of only one paper: Bürckert et al. [5] describe a holonic multi-agent system heuristic for a generalization of the PDPTWT in the context of long-distance transport. The authors take into account eight types of resource: driver, lorry with loading capacity, lorry without loading capacity, tractor, trailer, semi-trailer, chassis, and swap-body. Adequate combinations of these resources must be created to fulfil tasks.
Seminal works on *efficient feasibility tests* for insertion or local search procedures for different types of VRPs and PDPs are the ones by Savelsbergh [37, 38, 39], Kindervater and Savelsbergh [20], Funke et al. [13], Irnich et al. [18], Irnich [16, 17], Masson et al. [24], Vidal et al. [43], and Grangier et al. [14]. None of these, however, considers routing problems with trailers.

3 Adaptive Large Neighbourhood Search for the PDPTWT

Adaptive large neighbourhood search (ALNS) is a very widely and successfully used metaheuristic, in particular for, but not limited to, many different types of routing problem. ALNS was first used by Ropke and Pisinger [35] and extends the large neighbourhood search principle introduced by Shaw [40] by adding different removal and reinsertion operators and an adaptive operator selection scheme. Pisinger and Ropke [30] present a tutorial and a literature survey on (A)LNS. The basic idea of large neighbourhood search is to repeatedly perform the following steps. Given an incumbent solution, some of its elements are removed and reinserted to create a new solution that replaces the current incumbent if it either improves the best solution found so far or fulfils some other acceptance criterion.
The ALNS used for the computational experiments described in this paper follows the set-up described by Ropke and Pisinger [35]. All removal and reinsertion operators described by Ropke and Pisinger [35] (random, worst and Shaw removal, basic greedy, regret-2, -3, -4 and -M reinsertion) are applied, a roulette wheel procedure with adaptive weight adjustment is employed for selecting the removal and reinsertion operators in each iteration, and a simulated annealing acceptance criterion is used. In addition, the following three removal strategies are built into the ALNS. In the *arc frequency history removal* heuristic, proposed by Masson et al. [23], the aim is to remove tasks that seem to be at bad positions compared to the best known solutions. The heuristic keeps track of how often each arc (connection between two locations) appears in any one of the solutions contained in a fixed-size set composed of the best solutions found so far. In each ALNS iteration, if a solution enters or leaves this set, the frequencies of the arcs in this solution are incremented or decremented accordingly. When the arc frequency history removal heuristic is selected, a frequency value is computed for each task by summing up the frequencies of the arcs over which the pickup and the delivery locations of the task are reached and left in the current solution. Then,

with a certain amount of randomness as proposed by Ropke and Pisinger [35], the tasks with the lowest frequency values are removed. The *zero-split removal* heuristic, proposed by Parragh et al. [28], removes sequences of task locations where the vehicle is empty when reaching the first location and when leaving the last. Longer sequences are preferred, and the removed tasks are reinserted one by one. Finally, the *subroute removal* heuristic, as its name implies, removes entire subroutes, which are selected at random. 'Removing a subroute' means that all tasks with at least one location on the subroute are removed. The removed tasks are reinserted one by one in this heuristic, too.

The worst and Shaw removal heuristics exist in a static and a dynamic version. In the static versions, the removal criteria are computed anew only once in an ALNS iteration, in the dynamic versions, they are updated after each removal of a single task. The removal criterion for a task in the worst removal heuristic is the difference in the costs of the current solution with and without the task. The Shaw removal operator uses, for each pair of tasks, a relatedness measure that takes into account the distances between the pickup locations, the distances between the delivery locations, the overlap of the time windows of the pickup locations, the overlap of the time windows of the delivery locations, and the difference between the capacity requirements of the two tasks.

In lieu of the noise mechanism used by Ropke and Pisinger [35], the ALNS uses *insertion preference strategies*. In each iteration of a reinsertion heuristic, one of the following five strategies is randomly selected and applied before deciding which task to insert into which route: (i) make the insertion of tasks where the pickup location can be visited with a trailer more attractive; (ii) similar for tasks where this is not the case; (iii) make the insertion into single lorry routes more attractive; (iv) similar for LTC routes; (v) make it more attractive to insert tasks where the pickup location can be visited with a trailer into LTC routes. This is achieved by appropriately modifying the insertion costs of tasks into particular routes.

When, in an insertion step, the creation of a new subroute must be tested, which is necessary when a location not reachable by trailer is to be inserted directly after a location that is left with the trailer coupled, the ALNS selects a suitable PTL, but not necessarily the one closest to the task location in question. Similar to what is done in the removal heuristics, a certain degree of randomness is introduced.

4 Feasibility Tests

In the following, we propose techniques to test the temporal and capacitive feasibility of task insertions into routes performed by single lorries or LTCs in constant time, given appropriate auxiliary data computed in a preprocessing step. (In a slight abuse of terminology, 'amortized constant time' is abbreviated by 'constant time' here and in what follows.) As will be shown, the preprocessing to determine or update the necessary auxiliary data for a route to test time window as well as capacity feasibility takes time quadratic in the number of tasks fulfilled or locations visited on the route, but it is performed only once for a given solution. The resulting data are then used for all feasibility tests, i.e., for testing all potential insertion positions of all unplanned tasks. The routines are embedded in the ALNS metaheuristic described in the previous section. They could, however, also be used within other metaheuristic or local search approaches.

In this section, the following notation is used. Each task t from pickup location p to delivery location d is denoted by $t = (p, d)$ and has a capacity requirement $q^t > 0$, which means that q^t units of load must be picked up at p and $-q^t$ units must be delivered at d. The capacity requirement at each location u is denoted by q_u. Hence, $q_p > 0$ for each pickup location p, $q_d < 0$ for each delivery location d, and $q_u = 0$ for each vehicle depot or PTL u. Each location u has a single, hard time window $[a_u, b_u]$, $0 \le a_u \le b_u \le T$, where T is the length of the planning horizon. The depot locations have a time window of $[0, T]$. Each task location u has a unique service time (duration) s_u, and each PTL u has a decoupling duration (including a fixed time for a potential load transfer) of s_u^{dec} and a coupling duration of s_u^{coup}. For each pair (u, v) of locations, t_{uv} denotes the travel time from u to v. Each single lorry, each LTC lorry, and each trailer has a specified one-dimensional capacity, denoted by Q_k^l and Q_k^t respectively. For a single lorry k, $Q_k^t = 0$. The symbol

'==' serves as equality operator, '=' is the assignment operator, and '$x \mathrel{+}= y$' is used as shortcut for '$x = x + y$'.
The descriptions assume that feasibility of an insertion of a task $t = (p, d)$ into an existing route $r = (0, 1, \ldots, n-1, n)$, with p to be inserted directly after position (zero-based index of the route) h and d to be inserted directly after position i, is to be tested. If r is performed by an LTC and the trailer is attached when leaving h, a location triple $\tilde{p} = ptl_p \to p \to ptl_p$ corresponding to a new subroute is inserted after h; similar for i and d. ptl_p is a suitable trailer parking location; similar for d. Note that p, d, ptl_p, ptl_d, $\tilde{p}$, and $\tilde{d}$ are locations, whereas h and i are indices on a route. To simplify notation, when referring to a location visited at a certain position on a route, only the index is used: for example, the start of the time window of index i, i.e., of the ith location visited on a route, is denoted by a_i, and the travel time between index i and a to-be-inserted location v is denoted by t_{iv} etc.
Indices h and i indicate positions in the route *before* p and d are inserted. Hence, if $h == i$, then d is to be inserted directly after p, or, if a triple $\tilde{p} = ptl_p \to p \to ptl_p$ is to be inserted, directly after the triple. If, however, d cannot be reached with a trailer and p is left with the trailer attached or a triple $\tilde{p}$ is to be inserted, then a triple $\tilde{d} = ptl_d \to d \to ptl_d$ is inserted. In principle, if $h == i$ and p or d must be surrounded by a decouple-couple pair, it would also be possible to surround both p and d by one pair. This might be beneficial for instances where many pickups are close to their deliveries. For simplicity of exposition, we do not consider this additional possibility in the present paper. When this option is used, constant-time feasibility tests are just as well possible with the auxiliary data structures described in the following subsections; the formal description, though, is tedious. Moreover, in the course of an ALNS, configurations where it is beneficial that the pickup and the delivery of a task are surrounded by a decouple-couple pair will often be achieved automatically as a result of the removal steps.
Several consecutive subroutes by one LTC lorry at the same PTL are modelled by inserting a decouple-couple pair for each subroute. It is assumed that the fixed service times at PTLs are incurred also in such cases.

4.1 Time Windows

In this paper, we consider neither route duration constraints nor time-dependent costs and thus need not strive to minimize route duration. Under these conditions, it is optimal regarding feasibility to consider only as-early-as-possible schedules, i.e., to assume that a vehicle always leaves a location at the earliest possible point in time; this provides the maximum possible flexibility at subsequent positions on the vehicle's route.
Testing time-window feasibility of an insertion in linear time is trivial: the locations of the to-be-inserted task are tentatively inserted (including PTLs for decoupling and coupling, if necessary); the route is traversed, starting at the depot at time zero; travel, service and waiting times are added; finally, the resulting earliest possible starts of service are compared with the location time windows.
Testing time-window feasibility in constant time is a little more involved. To do so, Savelsbergh [39] introduced the concept of forward time slack (FTS). The FTS at a position on a route indicates by how much the earliest possible start of service at this position can be postponed without violating a time window at this or a subsequent position on the route. We adapt this idea to test the feasibility of the insertion of a pickup-and-delivery task (p, d) into a PDPTWT route $r = (0, \ldots, n)$ as follows.
First, note that a triple $\tilde{u} = ptl_u \to u \to ptl_u$ can be regarded as a *meta-location* or *segment* (cf. Irnich [16], Vidal et al. [43]) and handled as if it were a single location. Hence, whenever the insertion of a triple $\tilde{u}$ needs to be *tested* because the task location u cannot be reached with a trailer, the time window of the corresponding meta-location is tested. (However, when an insertion of a triple for a location u is to be actually *performed*, the sequence $ptl_u \to u \to ptl_u$ must be inserted, because the new subroute created by inserting the triple might be enlarged by an insertion of a task location in a later iteration.) The time window $[a_{\tilde{u}}, b_{\tilde{u}}]$ of a meta-location need be precom-

puted only once, before the start of the ALNS, for each task location u and each PTL ptl. This can be done by setting

$$a_{\tilde{u}} = \max(a_{ptl}, a_u - t_{ptl,u} - s^{dec}_{ptl}),$$
$$b_{\tilde{u}} = \min(b_u - t_{ptl,u} - s^{dec}_{ptl}, b_{ptl} - t_{u,ptl} - s_u - t_{ptl,u} - s^{dec}_{ptl}).$$

If $a_{\tilde{u}} > b_{\tilde{u}}$, then ptl cannot serve as parking location for visiting u. Otherwise, the service time $s_{\tilde{u}}$ of a meta-location $\tilde{u}$ is set to

$$s_{\tilde{u}} = s^{dec}_{ptl_u} + t_{ptl_u,u} + s_u + t_{u,ptl_u} + s^{coup}_{ptl_u}.$$

The travel times to and from a meta-location $\tilde{u}$ are those to and from ptl_u. The travel costs to $\tilde{u}$ are those to ptl_u for a lorry with its trailer plus those from ptl_u to u plus those from u to ptl_u, both for a lorry without its trailer. The travel costs from $\tilde{u}$ are those from ptl_u for a lorry with its trailer.

Second, the following auxiliary data are used:

- e_i: Earliest point in time at which service at index i can begin.
- w_i: Waiting time at index i, i.e., time period between arrival and beginning of service at i.
- w_{ij}: Cumulated waiting time between i and j, i.e., sum of waiting times at indices $i, \ldots, j$.
- f_i: Forward time slack at i.

These quantities are computed for each route in a preprocessing step as follows:

$$\begin{array}{lll}
e_0 = a_0; & e_i = \max(a_i, e_{i-1} + s_{i-1} + t_{i-1,i}); & i = 1, \ldots, n \\
w_0 = 0; & w_i = \max(0, a_i - e_i); & i = 1, \ldots, n \\
w_{00} = 0; & w_{0i} = w_{0,i-1} + w_i; & i = 1, \ldots, n \\
f_0 = b_0 - e_0; & f_i = \min(f_{i-1}, b_i - e_i + w_{0,i-1} + w_i); & i = 1, \ldots, n
\end{array}$$

The cumulated waiting time w_{ij} for $i > 0$ need not be stored but can be computed on-the-fly as $w_{ij} = w_{0j} - w_{0i}$. This means that the computation of the auxiliary data requires linear time in n. Nevertheless, as each w_{ij} may be used more than once in each ALNS iteration, it seems reasonable to compute and store these values in advance. Filling an appropriate data structure requires quadratic time in n. Thus, the overall time complexity of computing all auxiliary data for testing time-window feasibility of inserting a task into a route is quadratic in the number of tasks on the route. Given these data, time-window feasibility of an insertion can be tested as described in Algorithm 1 (cf. Masson et al. [24]).

Note that it is sufficient to execute lines 1–7 of Algorithm 1 only once for each h with a given PTL ptl_p. If TestTimeWindows returns false in line 7, it makes no sense to test further insertion positions for d with h as insertion position for p or $\tilde{p}$, because neither p nor $\tilde{p}$ can be inserted after h or later on r; hence, the next position for inserting p can be considered.

Due to the limited planning horizon, if a task location not reachable by trailer is to be inserted at a certain position on a main route, i.e., when a new subroute must be created, in principle all PTLs must be tested for whether an insertion at this position is possible. This, of course, increases the running time of an insertion heuristic. However, if only a subset of all PTLs is considered, an insertion heuristic may miss some feasible solutions, and the solution quality of the overall algorithm may deteriorate. The time window feasibility test described in Algorithm 1 receives as input a particular choice of PTL for the pickup and for the delivery location. Therefore, the test is exact in the sense that it will correctly consider the insertion of a specific triple $ptl_v \to v \to ptl_v$ feasible if and only if the insertion of this specific triple *is* feasible. If several PTLs shall be considered, Algorithm 1 must be embedded in a loop over these PTLs.

4.2 Capacities

Time-window tests are the same for single lorry as well as LTC routes: at each position on a route, the earliest start of service must lie within the time window of the respective location. By

Algorithm 1 TestTimeWindows($p, \tilde{p}, d, \tilde{d}, r, h, i$)

Input: Pickup-and-delivery task $t = (p, d)$
Route $r = (0, 1, 2, \ldots, n)$
Indices of insertion positions h, i with $0 \leq h \leq i \leq n-1$
Meta-locations $\tilde{p} = ptl_p \rightarrow p \rightarrow ptl_p$ and $\tilde{d} = ptl_d \rightarrow d \rightarrow ptl_d$ for p and d respectively, for a specific PTL ptl_p for p and a specific PTL ptl_d for d (only if r is performed by an LTC)

Result: Returns **true** iff inserting p or $\tilde{p}$ into r after index h and d or $\tilde{d}$ after i (or, iff $h == i$, after p or $\tilde{p}$) is feasible regarding all time windows, **false** otherwise

$u = p$
if p cannot be reached with trailer and trailer is currently attached
  $u = \tilde{p}$
// Test feasibility of inserting u after h
$e_u = \max(a_u, e_h + s_h + t_{h,u})$
if $e_u > b_u$
  return false
// Δ_{h+1} is the time shift at $h+1$, the increase of e_{h+1}, caused by inserting u
$\Delta_{h+1} = \max(0, e_u + s_u - e_{h+1} + t_{u,h+1})$
if $\Delta_{h+1} > f_{h+1}$
  return false
$v = d$
if d cannot be reached with trailer and trailer is currently attached
  $v = \tilde{d}$
// Test feasibility of inserting v after i
if $i > h$ // Delivery not directly after pickup
  $e_v = \max(a_v, a_i + \max(0, \Delta_{h+1} - w_{h+1,i}) + s_i + t_{i,v})$
  if $e_v > b_v$
    return false
  // Δ_{i+1} is the time shift at $i+1$ caused by inserting v
  $\Delta_{i+1} = \max(0, e_v + s_v - e_{i+1} + t_{v,i+1})$
  if $\Delta_{i+1} > f_{i+1}$
    return false
else // $i == h$, i.e., delivery directly after pickup
  $e_v = \max(a_v, e_u + s_u + t_{u,v})$
  if $e_v > b_v$
    return false
  $\Delta_{i+1} = \max(0, e_v + s_v + t_{v,i+1} - e_{i+1})$
  if $\Delta_{i+1} > f_{i+1}$
    return false
  return true

contrast, the presence of trailers requires additional capacity tests for LTC routes compared to single lorry routes. In this section, we first describe verbally what must be tested in linear- and constant-time capacity tests. Afterwards, we present our linear- and constant-time test routines.
At each position of *single lorry routes and main routes of LTCs*, the *total load balance*, which is the difference between the load picked up on the route so far minus the load delivered so far, must be less than or equal to the lorry plus the trailer capacity.
For capacity considerations on *subroutes*, the following two quantities are relevant:

- The *minimal lorry load at decoupling*, i.e., the minimal load that must inevitably be in the lorry upon leaving the decoupling location. This load is equal to the maximum of the following two values:
  - The difference between the total load balance at the decoupling location and the trailer capacity.
  - The sum of the capacity requirements incurred by the deliveries on the subroute whose pickups lie before the subroute. (This value is nonnegative, so that the minimal lorry load at the decoupling location is nonnegative as well.)
- The *subroute load balance* at each position, i.e., the difference between the sum of the load in the lorry at the start of the subroute plus the load picked up on this subroute so far minus the load delivered on this subroute so far. (The subroute load balance can be positive, zero, or negative.)

A subroute is capacity-feasible if and only if the first quantity is less than or equal to the lorry capacity and the absolute value of the second is less than or equal to the lorry capacity at each position.

4.2.1 Testing Capacities in Linear Time

To test capacity-feasibility of an insertion in linear time, we use the procedure detailed in Algorithm 2. For simplicity, the vehicle index k is omitted: Q^l and Q^t are used instead of Q^l_k and Q^t_k to denote the lorry and the trailer capacity.
Testing capacity in linear time for single-lorry routes is simple: the to-be-inserted task is tentatively inserted, one pass over the route is performed, and the capacity requirement at each position is added to the total load and compared with the lorry capacity (lines 2–6).
Testing capacity for LTC routes is not entirely straightforward even in linear time. As discussed above, it must be known at the start of a subroute how much load must be in the lorry to be able to perform the deliveries whose pickups are not on this subroute. This information is gathered in one forward pass over the route (lines 10–15). (In reality, it is of course not enough to have this amount of load in the lorry at the start of a subroute. It is also necessary to have the *right* commodities aboard the lorry, those that must be delivered on this subroute. This, however, has to be ensured by the driver. For algorithmic planning, it is sufficient to test whether enough loading capacity is available on the lorry.) The second pass (lines 19–37) then performs the actual capacity test on main routes and subroutes (total load at all positions, minimal load at decoupling positions, subroute load balance at all positions on subroutes).

4.2.2 Testing Capacities in Constant Time

To test the feasibility of the insertion of a pickup-and-delivery task (p,d) in constant time, the following data, computed for each route in a preprocessing step, can be used.

1. TrailerAttached: An array of boolean values. TrailerAttached[i] indicates whether or not the trailer is attached upon leaving (the location corresponding to) index i.
2. MaxTotalLoadOfSegment: A two-dimensional array of nonnegative integers. MaxTotalLoadOfSegment[i][*offset*] stores, for an index i on a route, the maximal load balance from the start of the route at any index from i up to and including $i+$*offset*. In particular, MaxTotalLoadOfSegment[i][0] stores the overall load picked up but not delivered yet from the start depot to and including the location at index i.

Algorithm 2 TestCapacityLinear(r, k)

Input: Route $r = (0, 1, 2, \ldots, n)$ with to-be-inserted task tentatively inserted, including decoupling and coupling locations where necessary, and capacity requirements $q_v \in \mathbb{Z}$, $v = 0, 1, 2, \ldots, n$
Vehicle k (single lorry or LTC) with capacities Q^l and Q^t; for single lorries, $Q^t == 0$

Result: Returns **true** iff lorry and, if applicable, trailer capacity of k are maintained at each index of r, **false** otherwise

```
1  TotalLoad = 0
2  if Q^t == 0 // Test for single lorries
3      for v = 0, 1, 2, ..., n
4          TotalLoad += q_v
5          if TotalLoad > Q^l
6              return false
7  else // Test for LTCs
8      LoadAtStartOfSubrouteToDeliver = array of integers of length n + 1, initialized to 0
9      IndexOfLastDecouple = 0
10     for v = 0, 1, 2, ..., n
11         if v corresponds to a decoupling location
12             IndexOfLastDecouple = v
13         if Trailer is not attached upon leaving v
14             if v corresponds to a delivery and associated pickup is before current subroute
15                 LoadAtStartOfSubrouteToDeliver[IndexOfLastDecouple] += (−1) · q_v
16     MinLorryLoadSinceLastDecouple = 0
17     MaxLorryLoad = 0
18     MaxLorryLoadSinceLastDecouple = 0
19     for v = 0, 1, 2, ..., n
20         TotalLoad += q_v
21         if TotalLoad > Q^l + Q^t
22             return false
23         MaxLorryLoad = min(TotalLoad, Q^l)
24         if v corresponds to a decoupling location
25             MinLorryLoadSinceLastDecouple =
26                 max(TotalLoad − Q^t, LoadAtStartOfSubrouteToDeliver[v])
27             if MinLorryLoadSinceLastDecouple > Q^l
28                 return false
29             MinLorryLoadSinceLastDecouple = max(MinLorryLoadSinceLastDecouple, 0)
30             MaxLorryLoadSinceLastDecouple = MaxLorryLoad
31         if Trailer is not attached upon leaving v
32             MinLorryLoadSinceLastDecouple = max(MinLorryLoadSinceLastDecouple + q_v, 0)
33             if MinLorryLoadSinceLastDecouple > Q^l
34                 return false
35             MaxLorryLoadSinceLastDecouple = min(MaxLorryLoadSinceLastDecouple + q_v, Q^l)
36             if q_v < 0 and MaxLorryLoadSinceLastDecouple < 0
37                 return false
38 return true
```

For example, consider the following route:

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Capacity requirement	0	+40	+10	0	+10	+20	–40	+5	–10	0	–10	–20	–5	0

This route contains one subroute, which starts at index 3 and ends at index 9, i.e., the zero value at index 3 corresponds to a decoupling process at some PTL, and the zero value at index 9 represents the associated coupling process at this PTL. The load balances from indices 2 to 6 are +50, +50, +60, +80, and +40; thus, MaxTotalLoadOfSegment[2][4] = +80. Moreover, MaxTotalLoadOfSegment[8][0] = +35.

3. TotalLoadDeliveredButNotPickedUpOnSubroute: An array of nonnegative integers. If i is an index corresponding to a decoupling location, TotalLoadDeliveredButNotPickedUpOnSubroute[i] stores the overall load delivered but not picked up on the respective subroute.
4. LoadBalanceFromStartOfSubroute: An array of integers. LoadBalanceFromStartOfSubroute[i] stores, for an index i on a subroute, the positive, negative or zero load balance from the start of the subroute up to and including i.
   In the above example route, LoadBalanceFromStartOfSubroute[7] = $-5 = 10 + 20 - 40 + 5$.
5. MaxLoadBalanceFromStartOfSubroute: A two-dimensional array of nonnegative integers. MaxLoadBalanceFromStartOfSubroute[i][*offset*] stores, for an index i on a subroute, the maximum of zero and the largest load balance from the start of the subroute to any index from i up to and including $i + \mathit{offset}$.
   In the above example, MaxLoadBalanceFromStartOfSubroute[6][2] = $0 = \max(0, -10, -5, -15) = \max(0, \text{LoadBalanceFromStartOfSubroute}[7])$.
6. IndexOfLastPrecedingDecouple: An array nonnegative integers. IndexOfLastPrecedingDecouple[i] stores the index where the last decoupling that precedes i on the route occurs.
7. OffsetOfNextCoupling: An array of nonnegative integers. OffsetOfNextCoupling[i] stores the number of positions on the route from i until the next index of a coupling process.

MaxTotalLoadOfSegment and MaxLoadBalanceFromStartOfSubroute can be filled using a nested forward pass, i.e., by iterating over all indices $j \geq i$ for each index i on the route. All other data structures described above can be filled or updated by passing through a route once. This means that all necessary preprocessing data for a route can be computed in quadratic time in the number of tasks on the route.

Given these data, the capacity feasibility of an insertion of a task $t = (p, d)$ into an existing route r, with p to be inserted directly after position (zero-based index of the route) h and d to be inserted directly after position i, can be tested as described in Algorithm 3. It is evident that the algorithm itself runs in constant time, i.e., its running time is independent of the number of tasks or the number of locations visited on route r.

Note that, to test the capacity constraints, it is irrelevant whether or not the pickup and/or the delivery location of the task to be inserted must be surrounded by a decouple-couple pair for insertion at the position in question, as decoupling and coupling processes have a capacity requirement of zero.

Note further that, similar to the situation in Algorithm 1, if TestCapacityConstant returns false from line 3 or line 16, it is unnecessary to consider further potential insertion positions for d for the current insertion position of p. Instead, the next position for inserting p can be considered. Hence, it is sufficient here to execute lines 2–16 of Algorithm 3 only once for each h.

5 Computational Experiments

5.1 Benchmark Instances

To this author's knowledge, there are no benchmark instances for the PDPTWT as studied in this paper. Therefore, a set of instances has been created to perform computational experiments with

Algorithm 3 TestCapacityConstant(p,d,r,h,i,k)

Input: Pickup-and-delivery task $t=(p,d)$ with capacity requirement $q>0$
Route $r=(0,1,2,\ldots,n)$
Indices of insertion positions h,i with $0\le h\le i\le n-1$
Vehicle k (single lorry or LTC) with lorry and trailer capacities Q^l and Q^t; for single lorries, $Q^t==0$

Result: Returns **true** iff inserting p or a triple $\tilde{p}=ptl_p\to p\to ptl_p$ into r after index h and d or a triple $\tilde{d}=ptl_d\to d\to ptl_d$ after i (or, iff $h==i$, after p or $\tilde{p}$) is feasible regarding lorry and trailer capacity, **false** otherwise

```
// Evaluate feasibility of insertion regarding total LTC capacity
if Q^l + Q^t < MaxTotalLoadOfSegment[h][i − h] + q
    return false
// Evaluate feasibility of insertion regarding lorry capacity
if i > h  // Delivery not directly after pickup
    // Evaluate feasibility of insertion of pickup
    if TrailerAttached[h] == false // Trailer not attached when leaving h
        ind = IndexOfLastPrecedingDecouple[h]
        MinLoadAtDecouple = max(MaxTotalLoadOfSegment[ind][0] − Q^t,
                                TotalLoadDeliveredButNotPickedUpOnSubroute[ind])
        LoadAfterPickup = MinLoadAtDecouple + LoadBalanceFromStartOfSubroute[h] + q
        offset = OffsetOfNextCoupling[h + 1]
        if h + OffsetOfNextCoupling[h] ≥ i
            offset = i − h
        if LoadAfterPickup + MaxLoadBalanceFromStartOfSubroute[h + 1][max(0, offset − 1)] > Q^l
            return false
    // Evaluate feasibility of insertion of delivery
    if TrailerAttached[i] == false
        if i − h ≥ OffsetOfNextCoupling[h] // Delivery not on same subroute as pickup
            ind = IndexOfLastPrecedingDecouple[i]
            MinLoadAtDecouple = max(MaxTotalLoadOfSegment[ind][0] − Q^t,
                                    TotalLoadDeliveredButNotPickedUpOnSubroute[ind])
            if MinLoadAtDecouple + MaxLoadBalanceFromStartOfSubroute[ind][i − ind] + q > Q^l
                return false
else // i == h, i.e., delivery directly after pickup
    if TrailerAttached[h] == false
        ind = IndexOfLastPrecedingDecouple[h]
        MinLoadAtDecouple = max(MaxTotalLoadOfSegment[ind][0] − Q^t,
                                TotalLoadDeliveredButNotPickedUpOnSubroute[ind])
        if MinLoadAtDecouple + LoadBalanceFromStartOfSubroute[h] + q > Q^l
            return false
return true
```

solution procedures. A well-known and widely used set of benchmark instances for pickup-and-delivery problems with time windows and without trailers has been proposed by Li and Lim [21] and is available at `www.sintef.no/projectweb/top/pdptw/li-lim-benchmark`. This set comprises six classes of instances, with 100, 200, 400, 600, 800, and 1,000 task locations, and thus with 50, 100, 200, 300, 400, and 500 tasks respectively. The instances have been derived from the Solomon instances for the vehicle routing problem with time windows (Solomon [41]), and in analogy to the original data, the Li and Lim instances are also partitioned into six classes LC1, LC2, LR1, LR2, LRC1, and LRC2 according to structural characteristics as follows: 'C' stands for geographically clustered tasks which, for the PDPTW and the PDPTWT, also means that the pickup and the delivery location of a task are close together; 'R' stands for geographically randomly distributed tasks; '1' stands for restrictive time windows so that only few tasks per route are possible; and '2' stands for less restrictive time windows and a longer planning horizon, which makes longer routes (routes covering more tasks) possible. Each instance has a homogeneous fleet, and start and end depot location of the vehicles coincide.

As pointed out by Derigs et al. [8], p. 544, some benchmark instances for vehicle routing problems with trailers are constructed such that there is no need to use lorry-trailer combinations at all, because the capacity of the lorry is high enough for transporting the entire demand and/or the time windows are so restrictive that a vehicle cannot serve many customers. This has also been observed when trying to modify the Li and Lim instances for use with trailers. Therefore, the benchmark instances for the PDPTWT have two vehicle classes: lorry-trailer combinations and single lorries. The single lorries have artificially high fixed cost, so that they are used only when necessary to ensure that all tasks are covered. Such cases can occur when the time windows of a task are so tight that there is not enough time to decouple the trailer to visit the pickup or the delivery task.

With this in mind, the Li and Lim instances have been adapted to the PDPTWT as follows:

- Every even-numbered location (as listed in the original Li and Lim instance file) is reachable by trailer, i.e., locations 0, 2, 4, 6...; the odd-numbered ones are not.
- Starting with location 0 (the depot), every second location that is reachable by trailer may be used for parking and transshipment; i.e., for locations 0, 4, 8, 12..., a PTL is created. This means that the number of PTLs is approximately half the number of tasks.
- As mentioned, the time windows of task locations are generally too short in the Li and Lim instances, so that no LTCs are used. Therefore, each original time window $[a_u, b_u]$ of a task location u is enlarged to $a_u = \max(0, a_u - TWShift)$ and $b_u = \min(b_u + TWShift, T)$, where $TWShift = \lfloor 100 + (AvgPickupTime + AvgDeliveryTime)/2 \rfloor$, and *AvgPickupTime* and *AvgDeliveryTime* respectively indicate the arithmetic mean of the service times at pickup and at delivery locations as indicated in the original files, rounded down to the nearest integer.
- The time windows of parking and transshipment locations are set to the complete planning horizon, i.e., to the time window of the depot. According to the author's practical experience, this is a mild and realistic assumption.
- The decoupling and coupling service times at PTLs are set to *AvgPickupTime* and *AvgDeliveryTime* respectively.
- The number of single lorries as well as the number of LTCs is considered unlimited.
- Single lorries are assigned a fixed cost of 1,000; LTCs have no fixed cost.
- As in the Li and Lim instances, Euclidean distances are used for travel times as well as travel costs. For LTCs, travel times and costs are the same whether or not the trailer is currently attached.
- Capacities of single and LTC lorries are set to the vehicle capacity specified in the respective original instance; trailer capacities are set to 150 % of the lorry capacity.

There is an arc between two locations u and v, i.e., a location v can be visited directly after a location u, unless $a_u + t^s_u + t_{uv} > b_v$, where $t^s_0 = 0$ for the depot location 0, $t^s_u = s_u$ for all task locations u, and $t^s_u = \min(AvgPickupTime, AvgDeliveryTime)$ for all PTLs u.

Table 1 shows the distribution of the number of instances of the different types and basic instance characteristics. Note that the number of tasks differs slightly between instances of the same size

Table 1: Instance characteristics

Size class	Type	No. instances	Average				Length planning horizon	Average				Lorry capacity
			No. tasks	No. locations	No. PTLs	No. arcs		Length time window	Pickup service time	Delivery service time	Capacity requirement	
100												
	LC1	9	53	135	27	16,309	1,236	584	80	90	19	200
	LC2	8	51	130	26	13,964	3,390	1,131	86	90	18	700
	LR1	12	53	134	27	17,633	230	209	9	10	15	200
	LR2	11	51	129	26	15,492	1,000	562	10	10	15	1,000
	LRC1	8	53	136	27	17,961	240	220	9	10	17	200
	LRC2	8	51	130	26	15,528	960	501	10	10	17	1,000
	All	56	52	132	27	16,222	1,100	513	32	34	16	543
200												
	LC1	10	105	266	53	63,713	1,351	628	81	90	18	200
	LC2	10	101	256	51	54,852	3,598	1,191	87	90	19	700
	LR1	10	105	264	53	63,138	634	341	9	10	17	200
	LR2	10	101	255	51	55,630	2,535	959	10	10	17	1,000
	LRC1	10	105	266	53	64,724	634	312	9	10	18	200
	LRC2	10	101	255	51	55,989	2,535	755	10	10	18	1,000
	All	60	103	260	52	59,674	1,881	698	34	37	18	550
400												
	LC1	10	210	527	106	248,962	1,501	652	82	90	18	200
	LC2	10	203	510	102	217,304	3,693	1,177	87	90	19	700
	LR1	10	209	525	105	241,818	804	384	9	10	18	200
	LR2	10	202	507	102	218,047	3,213	1,135	10	10	18	1,000
	LRC1	10	208	523	105	246,120	765	337	9	10	18	200
	LRC2	10	203	509	102	220,785	3,060	827	10	10	18	1,000
	All	60	206	517	104	232,172	2,173	752	34	37	18	550
600												
	LC1	10	315	791	158	551,278	1,496	653	81	90	18	200
	LC2	10	305	764	153	483,976	3,815	1,197	87	90	19	700
	LR1	10	314	788	158	512,648	1,206	476	9	10	18	200
	LR2	10	303	759	152	476,558	4,823	1,527	10	10	18	1,000
	LRC1	10	314	788	158	517,217	1,206	391	9	10	18	200
	LRC2	10	303	760	152	472,827	4,823	1,056	10	10	18	1,000
	All	60	309	775	155	502,417	2,895	883	34	37	18	550
800												
	LC1	10	419	1,051	210	946,974	1,676	676	82	90	18	200
	LC2	10	406	1,018	204	855,818	3,811	1,200	87	90	19	700
	LR1	10	418	1,049	210	871,608	1,688	573	9	10	18	200
	LR2	10	404	1,012	203	836,128	6,751	1,989	10	10	18	1,000
	LRC1	10	418	1,048	210	848,080	1,573	431	9	10	18	200
	LRC2	10	404	1,012	203	815,230	6,289	1,224	10	10	18	1,000
	All	60	412	1,032	207	862,306	3,631	1,016	34	37	18	550
1,000												
	LC1	10	525	1,314	263	1,420,904	1,824	684	82	90	18	200
	LC2	10	508	1,272	255	1,325,717	3,914	1,210	87	90	19	700
	LR1	10	523	1,309	262	1,325,967	1,925	617	9	10	18	200
	LR2	10	504	1,263	253	1,290,588	7,697	2,195	10	10	18	1,000
	LRC1	10	524	1,312	263	1,228,887	1,821	453	9	10	18	200
	LRC2	8	505	1,266	253	1,252,281	7,284	1,492	10	10	18	1,000
	All	58	515	1,290	258	1,309,291	3,967	1,095	35	38	18	534
All												
	All	354	267	670	134	497,857	2,617	828	34	36	18	546

class in the Li and Lim instances. Therefore, the values in the columns from 'No. locations' to 'No. Arcs' are averages, too. By construction of the instances, the column 'No. Tasks' also indicates the number of task locations reachable by trailer. Lorry capacities are the same for all instances of the same size class for each type.

5.2 Results

The code was programmed in C++ and compiled with Microsoft Visual Studio Enterprise 2017, Version 15.5.3. The experiments were run on a workstation with the Windows 10 Education operating system, an Intel Xeon E5-1660 v3 @ 3.00 GHz CPU, and 64 GB RAM in single-thread mode. The parameters used in the ALNS are listed in Table 3 in the Appendix.

To assess the relative performance of the linear- and the constant-time test, 10,000 ALNS iterations were performed with both tests for all instances of size classes 100, 200, and 400, i.e., those with at most 200 tasks. For the larger instances, computation times using the linear-time test became too long, so that, for size classes 600, 800, and 1,000, only the constant-time test was used. For the linear-time test, the time windows are tested together with the capacities in the loop of line 3 or 19 in Algorithm 2. The constant-time test first examines time window feasibility, then capacities. Aggregated results are shown in Table 2; detailed results by instance are given in Tables 4–9 in the Appendix.

The most important finding that can be read from Table 2 is that the speedup of the constant-time test compared to the linear one is considerable for all instance types and ranges from a factor of nine to a factor of 142, with an average of 38. This demonstrates that the effort of implementing the constant-time test is well justified.

Further insights that can be obtained from the data in Table 2 are:

- The larger the instance, the higher is the iteration number where the best solution was found.
- The number of routes in the best solution found can differ significantly between instances of the same size class and type.
- As LTC routes have no fixed cost, most routes are actually LTC routes. This also shows that the instances are a suitable test bed for routing problems with trailers (remember the comment on page 13).
- In particular for the larger instances with long planning horizon and wide time windows, the number of subroutes greatly exceeds the number of LTC routes, meaning that the average LTC route performs more than one subroute. Most PTLs are used only once.
- The running times for the instances with more tasks per route, i.e., fewer routes, are consistently higher than those for the other instances.
- The speedup for the LR and LRC instances increases with increasing instance size; for the LC instances, this is not the case.
- The speedup is significantly greater for the instances with fewer routes (classes with ‘2’).

6 Conclusions and Outlook

This paper has studied the PDPTWT, a routing problem which aims at fulfilling a set of transport tasks between pickup and delivery locations, subject to time window constraints and accessibility restrictions, by means of a fleet consisting of single lorries and lorry-trailer combinations. Procedures to test the temporal and capacitive feasibility of inserting a task into an existing route have been presented. Given adequate data computed in a preprocessing step, these procedures run in constant time. They have been embedded in an adaptive large neighbourhood search algorithm for the heuristic solution of the PDPTWT. A comprehensive set of benchmark instances has also been created. The results of computational experiments are presented which show significant speedups that can be realized with the constant-time feasibility test.

Topics for further research abound.

As the focus of the research presented here was on efficient feasibility testing, not on solution quality, many options exist regarding *algorithmic refinements* to improve solution quality of the ALNS. First of all, local and/or very large-scale neighbourhood search routines could be added, as done, e.g., by Derigs et al. [8] and Gschwind and Drexl [15]. Also, matheuristic components, e.g., solving a set-covering problem with all generated routes at the end of the ALNS, cf. Parragh and Schmid [29], Villegas et al. [45], could be helpful. Another refinement would be to add a splitting

Table 2: Aggregated computational results (minimum / average / maximum)

Size class	Type	Iteration where best solution was found	No. routes	No. LTC routes	No. sub-routes	No. PTLs used	Running time ALNS with constant-time test	Ratio running time linear / constant
100								
	LC1	1,850/5,505/9,249	10/ 11/ 11	10/11/ 11	10/10/ 11	6/ 8/10	42/ 44/ 47	10/ 11/ 12
	LC2	282/3,956/8,699	4/ 4/ 4	4/ 4/ 4	3/ 4/ 4	2/ 3/ 4	71/ 78/ 90	35/ 37/ 39
	LR1	4,794/8,111/9,850	11/ 11/ 11	11/11/ 11	10/11/ 12	6/ 7/ 9	39/ 42/ 45	9/ 9/ 10
	LR2	414/5,977/9,916	2/ 3/ 4	2/ 3/ 4	2/ 3/ 5	1/ 3/ 5	75/ 112/ 156	33/ 47/ 58
	LRC1	4,938/8,062/9,916	11/ 12/ 12	11/12/ 12	11/12/ 14	6/ 8/11	39/ 42/ 43	9/ 9/ 10
	LRC2	2,474/5,334/9,999	3/ 3/ 4	3/ 3/ 4	3/ 4/ 5	2/ 3/ 4	74/ 92/ 110	32/ 39/ 43
	All	282/6,276/9,999	2/ 7/ 12	2/ 7/ 12	2/ 7/ 14	1/ 5/11	39/ 68/ 156	9/ 25/ 58
200								
	LC1	5,116/8,530/9,980	20/ 21/ 21	20/21/ 21	20/21/ 22	12/14/16	148/ 157/ 169	11/ 12/ 12
	LC2	2,762/5,676/9,606	6/ 7/ 8	6/ 7/ 8	6/ 7/ 9	4/ 6/ 8	240/ 280/ 350	36/ 39/ 41
	LR1	4,497/7,663/9,645	8/ 10/ 11	8/10/ 11	8/12/ 16	7/ 9/11	175/ 198/ 229	19/ 22/ 26
	LR2	294/7,799/9,949	3/ 4/ 5	3/ 4/ 5	5/ 8/ 15	3/ 7/13	375/ 617/ 987	69/ 90/121
	LRC1	6,133/8,675/9,878	9/ 11/ 12	9/11/ 12	10/12/ 15	7/10/13	175/ 190/ 230	18/ 21/ 27
	LRC2	212/6,869/9,759	4/ 5/ 6	4/ 5/ 6	5/10/ 18	5/ 9/17	314/ 482/ 657	53/ 69/ 82
	All	212/7,535/9,980	3/ 9/ 21	3/ 9/ 21	5/12/ 22	3/ 9/17	148/ 321/ 987	11/ 42/121
400								
	LC1	7,750/9,320/9,998	36/ 39/ 42	36/39/ 42	35/40/ 42	19/25/29	468/ 494/ 521	11/ 11/ 12
	LC2	5,247/8,429/9,977	12/ 13/ 14	12/13/ 14	13/14/ 15	9/12/14	735/ 854/1,048	34/ 36/ 38
	LR1	6,366/8,456/9,987	15/ 18/ 21	15/18/ 21	17/21/ 26	13/18/20	538/ 630/ 763	19/ 24/ 31
	LR2	4,193/8,404/9,961	5/ 7/ 9	5/ 7/ 9	13/19/ 25	12/17/22	1,198/1,943/3,432	76/101/142
	LRC1	6,771/8,677/9,989	15/ 21/ 24	15/21/ 24	17/23/ 27	14/18/21	493/ 554/ 681	18/ 21/ 29
	LRC2	3,927/8,600/9,968	7/ 9/ 12	7/ 9/ 12	13/22/ 27	13/19/23	940/1,394/2,434	59/ 78/108
	All	3,927/8,648/9,998	5/ 18/ 42	5/18/ 42	13/23/ 42	9/18/29	468/ 978/3,432	11/ 45/142
600								
	LC1	8,262/9,523/9,958	57/ 61/ 64	57/61/ 64	55/61/ 65	34/40/45	778/ 820/ 868	
	LC2	8,330/9,425/9,964	19/ 21/ 22	19/21/ 22	20/24/ 32	17/22/28	1,092/1,259/1,505	
	LR1	3,047/8,285/9,981	18/ 25/ 31	17/24/ 31	23/34/ 47	21/30/37	1,010/1,163/1,452	
	LR2	7,062/8,658/9,915	7/ 10/ 12	7/10/ 12	28/42/ 60	25/35/48	2,216/3,584/5,803	
	LRC1	7,960/9,290/9,991	18/ 28/ 32	18/28/ 32	25/32/ 36	22/26/31	876/ 994/1,391	
	LRC2	5,731/9,177/9,995	8/ 12/ 17	8/12/ 17	29/44/ 61	25/37/53	1,742/2,394/4,247	
	All	3,047/9,060/9,995	7/ 26/ 64	7/26/ 64	20/40/ 65	17/32/53	778/1,702/5,803	
800								
	LC1	7,969/9,154/9,965	73/ 81/ 87	70/80/ 87	70/79/ 84	47/53/58	1,081/1,141/1,202	
	LC2	7,271/9,407/9,942	27/ 28/ 30	24/27/ 30	31/36/ 48	23/30/39	1,484/1,754/1,923	
	LR1	7,632/9,187/9,998	22/ 32/ 39	19/27/ 36	28/41/ 57	26/37/48	1,406/1,628/2,083	
	LR2	7,592/9,470/9,942	7/ 13/ 17	7/12/ 16	30/58/ 81	26/50/71	2,816/4,581/7,905	
	LRC1	8,637/9,521/9,989	26/ 40/ 47	24/36/ 40	35/43/ 50	32/37/43	1,116/1,286/1,743	
	LRC2	6,887/8,658/9,992	12/ 16/ 21	12/15/ 20	35/68/103	32/58/79	2,040/2,893/4,923	
	All	6,887/9,233/9,998	7/ 35/ 87	7/33/ 87	28/54/103	23/44/79	1,081/2,214/7,905	
1,000								
	LC1	5,994/8,963/9,999	92/103/110	88/95/103	88/94/108	62/68/78	1,310/1,477/1,597	
	LC2	7,300/9,098/9,988	34/ 36/ 38	32/35/ 36	40/48/ 58	33/42/52	1,956/2,196/2,581	
	LR1	8,048/9,504/9,997	27/ 40/ 50	22/32/ 40	37/54/ 68	34/48/55	1,822/2,151/2,800	
	LR2	8,723/9,477/9,910	10/ 16/ 21	10/14/ 17	43/69/ 99	42/62/83	3,275/5,497/1,002	
	LRC1	7,727/9,518/9,998	33/ 51/ 59	29/41/ 48	50/57/ 71	42/48/60	1,698/1,888/2,665	
	LRC2	8,358/9,499/9,934	14/ 20/ 24	13/18/ 21	56/89/118	50/75/97	2,937/4,035/6,626	
	All	5,994/9,338/9,999	10/ 45/110	10/40/103	37/68/118	33/56/97	1,310/2,834/1,002	
All								
	All	212/8,366/9,999	2/ 23/110	2/22/103	2/34/118	1/28/97	39/1,359/1,002	9/ 38/142

procedure based on dynamic programming that finds optimal PTLs for given routes, cf. Prins [31] and Villegas et al. [44].
Regarding *modelling extensions*, many additional practically relevant constraints could be taken into account. Two particularly interesting extensions are loading constraints such as last-in-first-out, and the impossibility of transferring load between an LTC lorry and its trailer. Of special relevance in connexion with constant-time feasibility tests are limits on route duration and on the time or the number of intermediate stops between the pickup and the delivery of a task. Load-dependent service times require an optimization of the load transfer amounts from lorry to trailer at decoupling and coupling locations, a considerable additional intricacy. Time-dependent costs (and route duration constraints, too) lead to the difficult situation that an as-early-as-possible schedule need no longer be optimal (Savelsbergh [39]), thus violating a fundamental assumption on which the feasibility tests described in the present paper are based.
Also *other variants of pickup-and-delivery problems*, such as one-to-many-to-one problems (also called vehicle routing problems with backhauls, Irnich et al. [19]), many-to-many, and simultaneous PDPs (Battarra et al. [2]), lend themselves to consider a fleet containing trailers.
Furthermore, in many pickup-and-delivery applications, the possibility or even the requirement to split tasks exists (cf. the survey by Drexl [11] and the more recent papers by Masson et al. [24] and Grangier et al. [14]). This means that a task $t = (p,d)$ can be decomposed into two subtasks or legs, (p,tl) and (tl,d) at transshipment locations tl. The legs of split tasks can be performed by different vehicles, and this creates an interdependence between routes: changes in one route may make one or several or all other routes infeasible. This interdependence requires a synchronization regarding time and load and, when trailers are considered, leads to the *PDPTWT with synchronization*.
Of course, all of the above extensions and variants can also be considered in a *dynamic and/or stochastic context*, where some information becomes known only after execution of a route plan has begun and/or some data are known only in the form of random variables, cf. Berbeglia et al. [4] and Flatberg et al. [12].
Finally, there is yet no *exact algorithm* for solving the PDPTWT. Computing optimal solutions to larger PDPTWT instances is surely a challenging but worthwhile endeavour.

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Appendix

The subsequent Table 3 specifies the parameter settings of the ALNS used for the computational experiments. The following Tables 4–9 present the detailed computational results for each of the benchmark instances with these settings. Table 2 was compiled based on these data. Euclidean distances were computed with full double precision, 10,000 iterations were performed, and the objective function values were rounded to three digits.

Table 3: ALNS parameter settings

Parameter	**Value**
Value for computing start temperature	5
Cooling rate	0.99975
Maximum number of iterations between update of performance statistics	100
Score 1	33
Score 2	9
Score 3	13
Score update factor	0.1
Absolute parameter for determining minimal number of tasks to be removed per iteration	30
Absolute parameter for determining maximal number of tasks to be removed per iteration	60
Relative parameter for determining minimal number of tasks to be removed per iteration	0.1
Relative parameter for determining maximal number of tasks to be removed per iteration	0.4
Randomization degree of worst removal heuristics	3
Randomization degree of Shaw removal heuristics	6
Randomization degree of arc frequency history removal heuristic	6
Randomization degree of zero split removal heuristic	6
Randomization degree of subroute removal heuristic	6
Distance weight parameter of Shaw removal heuristics	9
Time weight parameter of Shaw removal heuristics	3
Load weight parameter of Shaw removal heuristics	2
Number of solutions to be considered for arc frequency history removal	50

Table 4: Detailed computational results size class 100

Instance	Objective function value	Iteration where best solution was found	No. routes	No. LTC routes	No. subroutes	No. PTLs used	Running time ALNS with constant-time test (secs.)	Ratio running time linear / constant
lc101	970.473	6,326	11	11	10	9	42	10.7
lc102	944.132	6,396	10	10	10	9	44	10.4
lc103	1,015.599	8,318	11	11	10	8	42	10.6
lc104	951.527	3,546	10	10	10	9	46	11.6
lc105	986.038	5,441	10	10	10	7	43	11.1
lc106	972.096	5,194	11	11	10	8	44	10.8
lc107	1,013.618	1,850	11	11	11	9	45	10.7
lc108	998.764	9,249	11	11	11	10	46	10.7
lc109	959.115	3,228	10	10	10	6	47	11.3
lc201	691.886	3,004	4	4	4	4	71	37.7
lc202	716.538	7,761	4	4	4	4	79	35.5
lc203	725.034	282	4	4	4	3	80	36.1
lc204	659.023	8,699	4	4	4	4	90	36.1
lc205	661.215	4,978	4	4	3	3	71	35.7
lc206	664.417	1,897	4	4	4	2	76	38.1
lc207	679.853	1,363	4	4	3	3	77	37.5
lc208	676.502	3,667	4	4	3	3	79	38.8
lr101	1,138.993	7,753	11	11	11	7	43	9.6
lr102	1,171.374	4,794	11	11	11	6	45	10.0
lr103	1,223.400	4,819	11	11	10	7	40	9.1
lr104	1,090.987	8,552	11	11	10	6	41	9.6
lr105	1,136.728	9,054	11	11	10	6	43	9.6
lr106	1,152.816	9,850	11	11	11	9	40	9.4
lr107	1,216.950	9,236	11	11	11	8	40	9.2
lr108	1,162.082	6,254	11	11	10	6	39	8.8
lr109	1,127.926	9,309	11	11	12	6	43	9.9
lr110	1,109.671	9,538	11	11	12	6	40	9.6
lr111	1,207.751	9,313	11	11	12	9	45	9.5
lr112	1,162.763	8,856	11	11	11	7	42	9.7
lr201	1,041.385	7,809	4	4	4	3	75	33.2
lr202	1,071.902	7,699	3	3	3	3	91	40.6
lr203	969.654	7,084	3	3	5	5	107	45.6
lr204	846.491	4,767	2	2	2	2	142	57.6
lr205	992.878	7,503	3	3	4	3	98	43.1
lr206	975.402	9,916	3	3	5	5	103	44.3
lr207	884.184	3,222	2	2	2	1	139	57.1
lr208	734.643	414	2	2	2	1	156	57.1
lr209	928.528	2,202	3	3	4	4	104	44.4
lr210	938.639	5,383	3	3	3	2	109	48.5
lr211	833.859	9,747	3	3	3	2	104	45.4
lrc101	1,395.622	8,464	12	12	12	8	42	8.9
lrc102	1,505.317	9,916	12	12	12	11	42	8.7
lrc103	1,257.967	8,890	11	11	11	6	42	9.2
lrc104	1,197.082	6,581	11	11	11	8	43	9.6
lrc105	1,502.650	9,346	12	12	14	10	43	8.9
lrc106	1,440.623	4,938	12	12	12	9	41	8.8
lrc107	1,315.167	8,371	11	11	12	8	41	9.0
lrc108	1,284.733	7,992	11	11	11	7	39	9.2
lrc201	1,311.424	9,792	4	4	5	4	74	32.1
lrc202	1,192.597	5,102	3	3	3	2	82	36.5
lrc203	1,020.702	9,999	3	3	3	3	97	40.2
lrc204	819.319	3,083	3	3	3	3	108	41.0
lrc205	1,220.710	5,258	4	4	4	2	76	34.6
lrc206	1,175.625	2,474	3	3	4	4	93	39.9
lrc207	1,108.881	2,517	3	3	3	3	94	41.3
lrc208	885.852	4,444	3	3	3	3	110	42.7

Table 5: Detailed computational results size class 200

Instance	Objective function value	Iteration where best solution was found	No. routes	No. LTC routes	No. subroutes	No. PTLs used	Running time ALNS with constant-time test (secs.)	Ratio running time linear / constant
lc1_2_1	3,093.290	6,607	21	21	21	14	148	11.1
lc1_2_2	3,611.993	9,872	21	21	22	15	152	11.3
lc1_2_3	3,436.951	9,878	20	20	21	13	155	11.3
lc1_2_4	3,210.840	8,006	20	20	21	12	169	12.2
lc1_2_5	3,141.552	9,966	21	21	22	15	156	11.3
lc1_2_6	3,169.339	9,874	21	21	21	16	156	11.5
lc1_2_7	3,145.988	6,885	20	20	20	12	158	11.6
lc1_2_8	3,103.838	5,116	21	21	21	14	157	11.7
lc1_2_9	3,449.571	9,980	21	21	21	14	162	11.7
lc1_2_10	3,399.748	9,111	21	21	22	13	161	11.8
lc2_2_1	2,175.912	5,701	8	8	9	8	240	36.2
lc2_2_2	2,170.040	5,847	7	7	7	6	273	37.8
lc2_2_3	2,013.353	2,762	7	7	7	6	300	38.2
lc2_2_4	1,890.106	9,606	6	6	6	5	350	39.2
lc2_2_5	1,988.849	9,398	6	6	6	6	251	38.9
lc2_2_6	2,078.159	5,568	6	6	6	4	259	38.0
lc2_2_7	1,997.970	4,334	6	6	7	6	266	38.5
lc2_2_8	2,051.189	5,528	7	7	7	6	285	40.7
lc2_2_9	2,030.965	3,779	7	7	7	6	284	39.2
lc2_2_10	1,990.755	4,238	6	6	6	5	293	40.1
lr1_2_1	3,264.972	7,485	10	10	10	8	175	18.8
lr1_2_2	3,216.481	7,291	10	10	12	9	187	20.3
lr1_2_3	2,951.496	8,292	10	10	14	11	195	21.9
lr1_2_4	2,474.494	4,992	9	9	11	7	229	25.8
lr1_2_5	3,256.285	9,645	11	11	11	7	185	19.1
lr1_2_6	3,068.128	4,497	11	11	16	11	203	22.1
lr1_2_7	2,677.863	8,742	10	10	11	8	198	22.3
lr1_2_8	2,475.545	9,575	8	8	8	7	219	26.2
lr1_2_9	3,091.304	8,916	10	10	11	9	186	20.4
lr1_2_10	2,757.828	7,195	10	10	15	11	198	21.9
lr2_2_1	3,479.459	8,296	5	5	5	3	375	69.4
lr2_2_2	3,295.378	9,884	4	4	8	8	524	83.2
lr2_2_3	3,052.134	8,236	4	4	15	13	617	84.6
lr2_2_4	2,325.207	7,696	4	4	8	8	840	96.6
lr2_2_5	3,311.055	8,910	4	4	8	7	500	87.7
lr2_2_6	3,279.246	9,949	4	4	10	8	570	85.3
lr2_2_7	2,712.065	9,178	3	3	6	6	714	100.9
lr2_2_8	1,962.325	294	4	4	6	5	987	120.6
lr2_2_9	3,253.515	7,558	4	4	7	6	522	89.4
lr2_2_10	2,756.037	7,993	4	4	7	5	517	85.0
lrc1_2_1	2,892.339	6,809	12	12	13	13	175	17.8
lrc1_2_2	2,794.063	6,133	11	11	13	12	178	19.8
lrc1_2_3	2,663.717	8,898	9	9	13	11	203	22.8
lrc1_2_4	2,442.196	9,414	9	9	11	10	230	26.8
lrc1_2_5	3,102.716	9,166	11	11	11	9	180	18.9
lrc1_2_6	2,812.237	8,063	12	12	15	12	176	18.9
lrc1_2_7	2,826.656	9,688	11	11	11	7	187	20.3
lrc1_2_8	2,635.937	9,878	10	10	10	7	184	20.2
lrc1_2_9	2,593.870	9,457	10	10	13	10	185	20.7
lrc1_2_10	2,553.282	9,247	10	10	12	9	206	22.1
lrc2_2_1	2,861.418	8,416	6	6	12	9	314	53.3
lrc2_2_2	2,577.954	6,528	6	6	10	9	364	54.5
lrc2_2_3	2,393.727	212	5	5	7	6	500	72.4
lrc2_2_4	2,241.657	2,418	4	4	7	7	657	82.3
lrc2_2_5	2,859.009	8,398	5	5	18	17	466	72.1
lrc2_2_6	2,668.582	9,234	5	5	12	9	439	66.1
lrc2_2_7	2,544.252	9,759	5	5	9	9	434	61.1
lrc2_2_8	2,419.297	9,696	4	4	8	8	526	73.9
lrc2_2_9	2,272.824	6,722	4	4	5	5	527	75.0
lrc2_2_10	2,181.054	7,304	4	4	11	11	593	75.8

Table 6: Detailed computational results size class 400

Instance	Objective function value	Iteration where best solution was found	No. routes	No. LTC routes	No. subroutes	No. PTLs used	Running time ALNS with constant-time test (secs.)	Ratio running time linear / constant
lc1_4_1	8,157.242	9,894	40	40	42	29	468	10.9
lc1_4_2	8,326.408	9,606	39	39	39	24	490	11.4
lc1_4_3	8,181.272	9,487	38	38	40	24	511	11.9
lc1_4_4	7,998.630	9,611	36	36	35	23	521	12.3
lc1_4_5	8,452.623	8,748	42	42	42	28	472	11.0
lc1_4_6	7,965.205	7,750	40	40	40	25	482	11.2
lc1_4_7	8,101.512	9,001	40	40	42	28	491	11.3
lc1_4_8	8,065.265	9,911	39	39	39	26	483	11.1
lc1_4_9	8,606.076	9,197	38	38	39	27	506	11.4
lc1_4_10	8,063.120	9,998	37	37	39	19	516	11.5
lc2_4_1	4,634.553	8,424	13	13	13	9	735	33.9
lc2_4_2	4,718.921	9,331	13	13	14	9	830	34.9
lc2_4_3	4,732.656	5,581	13	13	15	13	943	35.3
lc2_4_4	4,832.596	5,247	13	13	15	12	1,048	35.8
lc2_4_5	4,793.259	9,977	14	14	14	13	760	34.8
lc2_4_6	4,330.294	8,316	12	12	13	11	810	36.2
lc2_4_7	4,455.024	9,622	13	13	14	14	830	36.6
lc2_4_8	4,542.666	8,399	13	13	13	11	843	38.0
lc2_4_9	4,796.374	9,561	13	13	15	12	863	36.5
lc2_4_10	4,542.475	9,835	13	13	15	13	880	37.8
lr1_4_1	7,600.421	9,901	21	21	24	20	538	18.9
lr1_4_2	7,122.111	8,661	20	20	20	19	589	21.4
lr1_4_3	6,293.120	9,987	17	17	19	18	650	24.5
lr1_4_4	5,453.027	9,352	15	15	17	13	756	28.9
lr1_4_5	7,197.859	6,366	20	20	26	20	544	22.8
lr1_4_6	6,924.309	9,903	18	18	20	16	614	22.3
lr1_4_7	6,248.194	6,972	17	17	23	19	660	26.0
lr1_4_8	5,421.693	7,130	15	15	18	15	763	30.5
lr1_4_9	6,931.926	9,867	19	19	20	17	574	20.8
lr1_4_10	6,615.693	6,418	18	18	23	20	616	22.9
lr2_4_1	8,661.925	9,136	9	9	15	12	1,198	76.2
lr2_4_2	7,568.071	9,942	8	8	19	16	1,528	83.5
lr2_4_3	6,753.730	4,193	8	8	25	22	1,981	97.3
lr2_4_4	5,311.441	9,961	6	6	18	16	2,868	122.0
lr2_4_5	7,686.022	8,490	8	8	17	16	1,400	93.2
lr2_4_6	6,735.141	4,596	8	8	20	17	1,678	93.4
lr2_4_7	6,054.436	8,945	6	6	15	13	2,238	114.8
lr2_4_8	5,226.712	8,953	5	5	13	12	3,432	142.5
lr2_4_9	7,196.682	9,888	8	8	24	22	1,508	94.0
lr2_4_10	6,732.745	9,932	8	8	24	22	1,599	93.7
lrc1_4_1	7,147.617	9,910	24	24	24	21	493	17.9
lrc1_4_2	6,410.331	6,771	21	21	24	19	546	20.6
lrc1_4_3	6,093.626	8,966	19	19	24	19	580	23.4
lrc1_4_4	5,340.786	9,989	15	15	17	14	681	29.1
lrc1_4_5	7,131.683	9,877	23	23	27	20	512	17.8
lrc1_4_6	6,542.183	9,589	22	22	24	16	515	19.3
lrc1_4_7	6,635.200	6,959	21	21	25	18	547	19.6
lrc1_4_8	6,390.169	9,938	22	22	23	21	550	19.7
lrc1_4_9	6,440.317	7,864	20	20	21	16	550	20.7
lrc1_4_10	5,877.081	6,906	19	19	20	15	565	22.2
lrc2_4_1	6,755.846	9,929	12	12	27	23	940	59.0
lrc2_4_2	6,372.873	5,437	10	10	22	20	1,184	69.9
lrc2_4_3	5,277.173	8,829	8	8	13	13	1,489	81.4
lrc2_4_4	4,656.752	9,893	7	7	24	22	2,434	108.1
lrc2_4_5	6,432.971	9,520	10	10	20	19	1,079	65.4
lrc2_4_6	6,318.373	3,927	10	10	27	21	1,094	67.4
lrc2_4_7	5,902.167	9,607	9	9	21	17	1,230	73.5
lrc2_4_8	5,560.296	9,767	8	8	21	18	1,424	81.6
lrc2_4_9	5,289.767	9,968	8	8	15	15	1,467	85.1
lrc2_4_10	5,264.808	9,125	8	8	25	22	1,598	88.1

Table 7: Detailed computational results size class 600

Instance	Objective function value	Iteration where best solution was found	No. routes	No. LTC routes	No. subroutes	No. PTLs used	Running time ALNS with constant-time test (secs.)
lc1_6_1	16,466.091	9,762	62	62	62	43	778
lc1_6_2	16,521.563	9,763	61	61	61	37	799
lc1_6_3	16,629.909	8,754	58	58	58	38	826
lc1_6_4	16,045.254	9,958	57	57	55	34	857
lc1_6_5	17,075.448	9,886	64	64	61	45	789
lc1_6_6	17,092.862	9,858	64	64	65	44	820
lc1_6_7	16,287.753	9,772	61	61	59	36	818
lc1_6_8	17,413.197	9,934	62	62	63	43	815
lc1_6_9	18,047.010	9,281	62	62	63	35	834
lc1_6_10	17,327.674	8,262	59	59	61	41	868
lc2_6_1	10,207.809	9,964	22	22	25	22	1,092
lc2_6_2	9,563.355	8,330	19	19	21	17	1,204
lc2_6_3	9,454.697	9,235	20	20	26	22	1,352
lc2_6_4	8,949.587	9,736	20	20	20	19	1,505
lc2_6_5	9,761.939	9,942	20	20	23	22	1,137
lc2_6_6	9,809.889	9,927	22	22	32	28	1,189
lc2_6_7	9,437.012	9,493	21	21	25	23	1,216
lc2_6_8	9,172.899	8,737	21	21	24	20	1,263
lc2_6_9	9,478.359	9,545	21	21	24	23	1,263
lc2_6_10	8,853.279	9,338	20	20	23	21	1,367
lr1_6_1	16,971.485	9,266	31	31	47	37	1,010
lr1_6_2	17,411.229	5,466	27	25	34	30	1,074
lr1_6_3	14,309.551	9,961	24	24	31	28	1,229
lr1_6_4	12,432.560	8,843	18	17	23	21	1,406
lr1_6_5	16,143.579	7,255	28	28	37	34	1,016
lr1_6_6	16,917.078	9,557	25	23	34	30	1,119
lr1_6_7	13,302.893	9,981	23	23	28	26	1,215
lr1_6_8	11,141.020	9,976	19	19	33	31	1,452
lr1_6_9	15,254.064	9,497	28	28	36	30	1,021
lr1_6_10	16,886.003	3,047	26	25	35	34	1,088
lr2_6_1	18,358.684	8,917	12	12	42	34	2,216
lr2_6_2	15,947.769	8,049	12	12	45	37	2,843
lr2_6_3	14,063.910	7,062	10	10	41	37	3,651
lr2_6_4	11,140.270	9,007	7	7	28	25	5,599
lr2_6_5	17,091.142	7,637	11	11	60	48	2,596
lr2_6_6	15,925.030	9,745	10	10	43	32	3,230
lr2_6_7	13,519.317	9,566	9	9	45	38	4,105
lr2_6_8	10,320.749	7,862	7	7	29	26	5,803
lr2_6_9	15,666.296	9,915	10	10	37	29	2,819
lr2_6_10	15,595.795	8,820	11	11	54	45	2,975
lrc1_6_1	13,855.237	9,714	32	32	35	25	876
lrc1_6_2	12,973.273	9,991	29	29	35	28	969
lrc1_6_3	12,659.233	9,834	25	25	34	30	1,117
lrc1_6_4	9,902.372	9,652	18	18	25	22	1,391
lrc1_6_5	13,645.705	7,960	32	32	36	31	898
lrc1_6_6	13,554.014	8,787	30	30	32	24	901
lrc1_6_7	12,950.530	8,703	29	29	35	27	933
lrc1_6_8	13,145.138	8,813	28	28	30	26	931
lrc1_6_9	13,227.627	9,645	28	28	29	24	951
lrc1_6_10	12,424.974	9,805	26	26	32	23	977
lrc2_6_1	14,628.817	9,995	17	17	61	53	1,742
lrc2_6_2	13,179.837	9,591	14	14	59	50	2,131
lrc2_6_3	11,013.262	9,799	11	11	32	27	2,705
lrc2_6_4	9,414.274	9,639	8	8	29	25	4,247
lrc2_6_5	13,483.235	9,753	15	15	39	32	1,802
lrc2_6_6	13,981.647	9,152	13	13	47	39	1,865
lrc2_6_7	12,920.265	8,311	11	11	34	28	2,220
lrc2_6_8	12,120.200	9,915	12	12	43	40	2,291
lrc2_6_9	13,734.873	5,731	11	11	50	41	2,321
lrc2_6_10	12,396.543	9,886	10	10	44	35	2,618

Table 8: Detailed computational results size class 800

Instance	Objective function value	Iteration where best solution was found	No. routes	No. LTC routes	No. subroutes	No. PTLs used	Running time ALNS with constant-time test (secs.)
lc181	29,779.459	9,751	84	84	84	58	1,081
lc182	34,001.982	7,969	81	79	76	56	1,129
lc183	31,269.812	8,397	78	78	79	53	1,148
lc184	31,909.582	8,508	73	70	70	47	1,202
lc185	32,090.767	9,405	87	87	84	55	1,136
lc186	30,526.773	9,015	84	84	83	57	1,152
lc187	30,846.858	8,924	82	82	80	51	1,123
lc188	31,051.107	9,965	82	82	83	54	1,141
lc189	31,594.426	9,834	79	79	75	50	1,148
lc1810	31,669.092	9,775	79	79	79	52	1,147
lc281	15,159.593	9,597	28	28	32	25	1,484
lc282	16,926.027	9,467	30	30	48	39	1,628
lc283	19,443.635	8,930	28	24	33	32	1,727
lc284	17,848.417	9,571	27	25	32	26	1,922
lc285	14,943.285	9,821	27	27	31	23	1,631
lc286	15,451.011	9,651	29	29	43	34	1,766
lc287	15,270.417	9,942	27	26	32	26	1,835
lc288	14,743.008	9,902	28	28	31	28	1,823
lc289	14,756.891	9,916	28	28	32	27	1,804
lc2810	15,605.859	7,271	29	29	43	36	1,923
lr181	41,949.098	9,001	38	27	48	42	1,431
lr182	32,628.581	9,008	37	32	48	45	1,513
lr183	30,878.496	9,914	29	23	39	35	1,686
lr184	21,989.100	9,897	22	19	34	34	2,082
lr185	32,183.407	9,916	39	36	57	48	1,406
lr186	33,190.601	9,998	35	28	36	30	1,515
lr187	25,346.191	9,803	29	27	35	33	1,684
lr188	20,062.252	7,632	23	21	28	26	2,083
lr189	34,384.747	8,366	37	31	48	43	1,406
lr1810	31,676.044	8,330	34	27	41	35	1,471
lr281	33,711.230	7,592	17	16	81	71	2,816
lr282	28,988.585	9,681	13	13	59	55	3,589
lr283	23,193.891	9,942	12	11	51	43	4,739
lr284	18,490.427	9,455	8	8	34	31	7,834
lr285	29,125.350	9,666	15	14	70	58	3,105
lr286	24,148.337	9,663	14	14	58	47	3,967
lr287	22,686.727	9,918	12	12	62	56	5,224
lr288	17,010.234	9,518	7	7	30	26	7,905
lr289	27,858.035	9,842	13	13	66	54	3,235
lr2810	25,980.059	9,423	14	14	70	54	3,395
lrc181	33,874.714	8,637	47	40	48	40	1,116
lrc182	29,331.441	9,941	44	39	43	35	1,230
lrc183	24,651.073	8,960	36	33	42	35	1,411
lrc184	19,563.444	9,296	26	24	35	32	1,743
lrc185	32,054.570	9,468	45	39	43	38	1,179
lrc186	30,576.310	9,699	44	38	42	38	1,225
lrc187	29,794.101	9,970	43	39	50	43	1,208
lrc188	26,477.220	9,989	39	36	45	37	1,268
lrc189	22,577.839	9,984	38	38	46	37	1,224
lrc1810	24,871.446	9,263	37	34	39	35	1,258
lrc281	25,832.234	6,887	21	20	75	59	2,040
lrc282	24,480.590	9,191	17	14	51	50	2,551
lrc283	19,833.313	9,929	16	15	53	44	3,133
lrc284	14,549.916	9,218	12	12	35	32	4,923
lrc285	23,944.816	7,576	19	19	103	79	2,408
lrc286	24,865.120	8,616	17	15	87	76	2,401
lrc287	22,520.066	6,938	16	16	76	64	2,543
lrc288	21,990.194	8,369	16	15	84	71	2,848
lrc289	19,734.446	9,866	15	15	77	68	2,864
lrc2810	19,399.705	9,992	13	13	39	39	3,223

Table 9: Detailed computational results size class 1,000

Instance	Objective function value	Iteration where best solution was found	No. routes	No. LTC routes	No. subroutes	No. PTLs used	Running time ALNS with constant-time test (secs.)
lc1101	65,073.718	9,695	108	94	89	64	1,310
lc1102	67,649.754	7,571	103	90	93	70	1,369
lc1103	55,315.955	9,803	95	90	93	69	1,516
lc1104	53,303.065	7,549	92	88	88	63	1,521
lc1105	61,452.807	9,950	105	95	91	65	1,426
lc1106	59,887.447	9,518	110	103	96	68	1,465
lc1107	55,160.075	9,995	105	102	98	72	1,494
lc1108	63,357.039	9,999	105	96	94	69	1,512
lc1109	61,419.643	5,994	108	103	108	78	1,557
lc11010	54,890.505	9,554	97	93	90	62	1,597
lc2101	22,527.689	9,988	35	34	40	33	1,956
lc2102	23,792.805	7,876	36	35	51	46	2,169
lc2103	23,031.139	9,819	34	34	48	42	2,321
lc2104	25,118.021	9,545	34	32	46	39	2,581
lc2105	24,710.473	9,475	37	35	41	37	2,073
lc2106	23,694.035	9,704	36	35	46	40	1,995
lc2107	25,279.518	9,221	38	36	58	52	2,179
lc2108	25,028.226	7,300	37	35	49	42	2,237
lc2109	25,348.545	8,748	37	36	55	50	2,259
lc21010	21,934.962	9,304	36	36	44	35	2,185
lr1101	59,691.009	9,975	50	35	61	51	1,822
lr1102	48,538.500	9,997	45	39	64	53	1,909
lr1103	44,318.348	9,671	37	29	53	48	2,215
lr1104	32,194.887	9,130	27	22	37	34	2,739
lr1105	56,018.817	9,783	49	40	68	55	1,837
lr1106	45,459.665	9,881	42	35	49	43	2,026
lr1107	41,921.754	9,202	35	26	48	43	2,305
lr1108	35,527.347	9,499	28	23	48	46	2,800
lr1109	48,613.159	9,857	46	39	53	50	1,888
lr11010	47,691.898	8,048	41	32	59	52	1,968
lr2101	51,690.254	9,868	21	17	99	83	3,275
lr2102	45,995.484	9,789	18	16	70	64	4,190
lr2103	36,607.173	9,896	16	13	43	42	5,416
lr2104	27,180.136	9,254	12	12	59	54	9,069
lr2105	46,941.618	9,595	18	15	77	67	3,775
lr2106	41,616.661	8,730	17	15	70	61	4,741
lr2107	34,561.985	9,174	14	12	64	59	6,095
lr2108	26,083.772	9,832	10	10	50	46	10,024
lr2109	43,679.631	8,723	18	15	86	72	4,052
lr21010	41,890.994	9,910	16	13	75	72	4,336
lrc1101	56,643.406	9,370	59	44	57	42	1,698
lrc1102	48,314.071	9,827	56	48	71	60	1,814
lrc1103	39,926.935	9,798	46	39	51	45	2,051
lrc1104	32,640.504	7,727	33	29	50	45	2,665
lrc1105	54,919.086	9,631	58	47	58	44	1,717
lrc1106	55,289.112	9,903	55	39	55	50	1,708
lrc1107	46,741.471	9,827	50	39	51	42	1,758
lrc1108	50,108.697	9,452	51	39	61	52	1,790
lrc1109	46,641.948	9,651	50	39	52	45	1,803
lrc11010	39,637.235	9,998	47	42	59	51	1,871
lrc2101	39,684.964	9,595	24	20	101	86	2,937
lrc2102	37,322.263	9,847	23	20	102	81	3,435
lrc2103	30,658.094	9,759	19	18	80	69	4,325
lrc2104	25,217.808	9,934	14	13	56	50	6,626
lrc2105	38,071.655	9,897	21	16	88	74	3,442
lrc2106	33,867.241	9,521	21	21	118	97	3,379
lrc2107	33,795.104	9,081	18	16	83	73	3,720
lrc21010	33,794.739	8,358	18	16	85	71	4,416