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Rubinstein Bargaining with Other-Regarding Preferences

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Abstract

While classic bargaining theory abstracts from other-regarding motives, bargaining processes often take place among parties who care about each other's payoff. In this paper, I analyze how other-regarding preferences affect the outcome, duration, and use of means to harm the other in reference to a Rubinstein bargaining game. It is found that agents regarding each other's payoff negatively will reach less equal outcomes, take longer to reach this outcome and are more likely to harm each other if they have means available to do so.

1. *Introduction*

Episodes of what might be called true conflict[1] often end in, or are interrupted by, a process of bargaining over the initially disputed claims, possibly also over additional resources. The negotiations between Palestinians and Israelis in Camp David or, more recently, the successful peace talks in Colombia may serve as but two illustrative examples offered by history.

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[1]Cases of conflict of interest are abundant, and mostly they do not pose a serious threat for peace - think of the conflicts of interest between a buyer and a seller in a market economy. By using the term 'true conflict', I intend to refer to conflicts of interest in which either one or both sides of a dispute have used means to harm the other party, violent conflicts offering the most vivid example.

In general, the insight of the parties involved to try to solve the conflict by bargaining instead of possibly more violent means can be considered a valuable first step. What might complicate the matter, however, is the fact that parties may well have developed other-regarding preferences over the course of the conflict. Typically, these can be assumed negative in nature, i.e. the parties mutually "dislike" each other. Such negative other-regarding preferences may hinder finding an agreement even if parties have agreed to do so without using violent means. In this paper, I analyze the effects of other-regarding preferences on the process and the outcome of a Rubinstein bargaining game.

2. *Conflict as Breakdown of Bargaining: The Baseline Model*

Assume that two agents, i and j, with other-regarding preferences bargain over the division of a given resource according to the alternating offers bargaining model introduced by Rubinstein (1982). Without loss of generality, the overall 'value' of this resource is normalized to unity. For the sake of representation, I refer to this value as monetary value. [2] Utilities may then take the following form:

$$u_i = \pi_i + \alpha_i \pi_j, \tag{2.1}$$

where π_i represents agent i's monetary payoffs, and $\alpha_i \in [-1, 1]$ the weight with which he incorporates agent j's monetary payoff π_j in his own utility considerations. This weight α_i may depend on several factors, such as reciprocity, altruism, inequality aversion or similar other-regarding motives. Any future period is discounted by the discount factors δ_i and δ_j by players i and j, respectively. Say that player i makes the initial offer, and will continue to make offers in any odd numbered period as long as the game proceeds. Whenever player j rejects an offer, he will in any even numbered period be able to make counter offers.

Call the maximum amount player i will be able to assure himself in any odd numbered period x. This amount yields him a level of utility of

[2] See Kohler (2013) for a model incorporating envy in an alternating-offers setting via preferences including inequality aversion as in Fehr and Schmidt (1999). The present analysis differs in that it takes a more general approach to other-regarding preferences, allowing not only for envy or inequality aversion, but other forms such as reciprocity as well. Further, I include these preferences not only in the standard model of alternating offers, but also in the extension with punishment options.

$u_i = x+\alpha_i(1-x)$.[3] Consequently, $\delta_i u_i(x)$ will be his fallback option in the preceding period.[4] Knowing this, the maximum amount j will be able to enforce in this period will assure him a level of utility of $u_j = 1 - \delta_i(x + \alpha_i(1-x)) + \alpha_j x$. The discounted value of this utility level will constitute his fallback option in the preceding period, such that player i will have to offer player j at least this value, i.e. $\delta_j(1 - \delta_i(x + \alpha_i(1-x)) + \alpha_j x)$. Hence, the maximum amount x player i can assure himself satisfies the following equation:

$$x + \alpha_i(1-x) = 1 - \delta_j(1 - \delta_i(x + \alpha_i(1-x)) + \alpha_j x) + \alpha_i(1-x). \quad (2.2)$$

Solving for x yields the offer i will make when it's his turn to offer. Call this offer x^0_{ii}, the first subscript indicating the player who offers, the second subscript the player who receives the respective share. x^0_{ii} then is:

$$x^0_{ii} = \frac{1 - \delta_j - \delta_i\delta_j\alpha_i}{1 - \delta_i\delta_j + \delta_i\delta_j\alpha_i + \delta_j\alpha_j}. \quad (2.3)$$

Assuming rational agents, this initial offer will be accepted by player j in the initial period, and no costs of delay will be incurred. By analysis of (2.3), the first result follows:

Result 1 If $x^0_{ii} > 0.5$, the equilibrium offers will be more equal, the larger the other-regarding components in the players' preferences, *ceteris paribus.*

Proof. Note that for $x^0_{ii} > 0.5$, it has to hold that $1 + \delta_i\delta_j + \delta_j\alpha_j > 3\delta_i\delta_j\alpha_i + 2\delta_j$. This implies "sufficiently large discounting" and / or "sufficiently small α_i". The first-order partial derivative of x^0_{ii} with respect to α_i can be reformulated to:

$$\frac{\partial x^0_{ii}}{\partial \alpha_i} = \frac{-\delta_i\delta_j(2 - \delta_i\delta_j + \delta_j\alpha_j - \delta_j)}{(1 - \delta_i\delta_j + \delta_i\delta_j\alpha_i + \delta_j\alpha_j)^2} \quad (2.4)$$

[3] For the sake of completeness, note that the utility levels under the given assumptions are constrained to fall within the unit interval.

[4] I here apply the derivation of the equilibria following Shaked and Sutton (1984). A more extensive derivation is offered in the appendix, which also contrasts the outcomes of different versions of the Rubinstein bargaining game.

Given the assumed range of parameter values, it follows that (2.4) is negative.[5] That is, an increase in i's positive other-regardingness of j will always lead him to make lower offers in equilibrium. It can also easily seen from (2.3) that increases in α_j lead to increases in equilibrium offers. The more j takes i's payoff (positively) into account, the more will i be able to set through in equilibrium. □

It can readily be seen from (2.3) that, as $\alpha_i, \alpha_j \to 0$ the solution converges to the standard solution in the alternating offers game, namely $x_{ii}^{AO} = \frac{1-\delta_j}{1-\delta_i\delta_j}$. Further, increases in α_i will lead to lower equilibrium offers, ceteris paribus. Intuitively, the more one player takes the monetary payoffs of the other (positively) into account, the less he will demand for himself. The reason is that he would otherwise (partly) "hurt himself" by "harming the other".

3. *Conflict as Bargaining: Introducing Punishment Options*

Bargaining situations are sometimes characterized by punishment options. That is, either of the two bargaining parties may be in a position to impose costs on the other, possibly at a cost to himself. Strikes in the process of wage bargaining are but one example, warfare another (see Fernandez and Glazer (1991) for an analysis of the former, and Slantchev (2003) for an analysis of the latter cases). The model introduced in the previous section can readily be adjusted to take these costs into account.

Say that player i may impose costs of $c_{ij} > 0$ on his opponent if his offer is rejected, the first subscript again indicating the player who may 'choose', the second the player who is affected. By choosing to inflict costs on j, i will incur costs of $c_{ii} > 0$. Similarly, j may impose costs of $c_{ji} > 0$ on i, in which case he will have to bear costs of $c_{jj} > 0$. Figure 3.1 illustrates the structure of the alternating offers game with punishment.[6]

Note that the introduction of punishment is accompanied by possible

[5] To see this, note that the numerator is negative iff $2 - \delta_j - \delta_i\delta_j + \delta_j\alpha_j > 0$. This condition can be rearranged to $\delta_i < \frac{2-\delta_j+\delta_j\alpha_j}{\delta_j}$. Because all parameters are defined over the unit interval, the right-hand side of this expression will be larger than 1, which implies that the condition holds for the complete defined range of δ_i.

[6] Note that the structure presented in 3.1 allows both players to punish even if they have rejected an offer in the very same period. This option will not be chosen by rational players, however, as Avery and Zemsky (1994: 158) have argued; punishment will only be chosen after *the other player* rejected an offer.

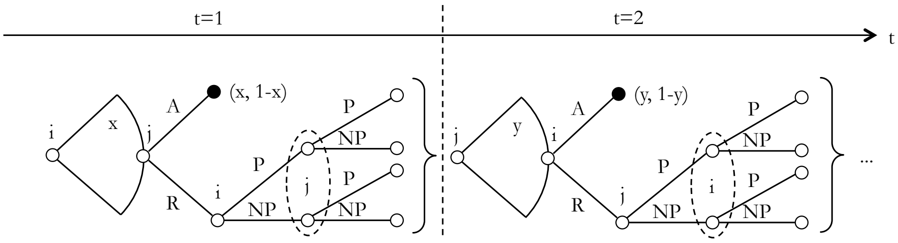

Figure 3.1: The Basic Structure of Alternating Offers Game With Punishment

inefficiencies. In case either player indeed chose to punish, the overall amount to be divided can only diminish. Applying the same logic as before, i will now yield a utility level of:

$$u_i = x + \alpha_i(1 - x) - c_{ii} - c_{ji}.^7 \tag{3.1}$$

Accordingly, if i is the first to make an offer, the share he will be able to assure himself now is given by:

$$x_{ii}^P = \frac{1 - \delta_j + \delta_i\delta_j\alpha_i - \delta_i\delta_j(c_{ii} + c_{ji}) + (c_{ij} + c_{jj})}{(1 - \delta_i\delta_j + \delta_i\delta_j\alpha_i + \delta_j\alpha_j)}. \tag{3.2}$$

The share x_{ii}^P for i corresponds to the share $x_{ij}^P = (1 - x_{ii}^P)$ for player j. Hence,

$$x_{ij}^P = \frac{\delta_j - \delta_i\delta_j + \delta_j\alpha_j + \delta_i\delta_j(c_{ii} + c_{ji}) - (c_{ij} + c_{jj})}{1 - \delta_i\delta_j + \delta_i\delta_j\alpha_i + \delta_j\alpha_j} \tag{3.3}$$

It can be seen from equations (3.2) and (3.3) that the more costs either player can impose on his opponent, the higher will be his share of the surplus to be divided, all else equal. Also, the more expensive it is for a player to impose a given cost on his opponent, the lower will his share be, again all else held constant. As all cost components approach zero, (3.2) converges to (2.3).

Either player may now find it profitable to indeed punish his opponent after rejection of his offer. He may do so if be believes that his letting the rejection go unpunished will lead to another equilibrium where his payoff is lower.

[7] I find it reasonable to assume that the players' other-regardingness does not include costs of punishment, but only the monetary payoffs from the offer per se. That is, if i punishes j, i will incur costs of c_{ii}. But the other-regarding component of j is $\alpha_j x$, and not $\alpha_j(x - c_{ii})$.

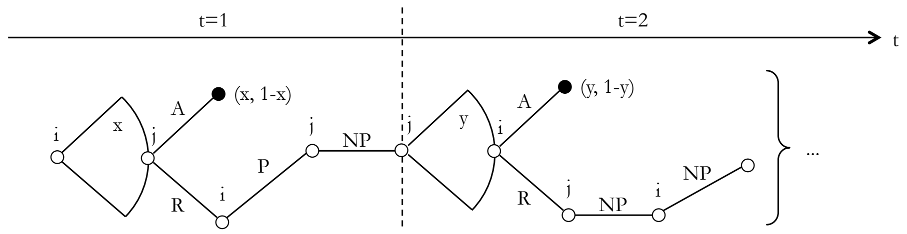

Figure 3.2: The Structure of *Regime i*

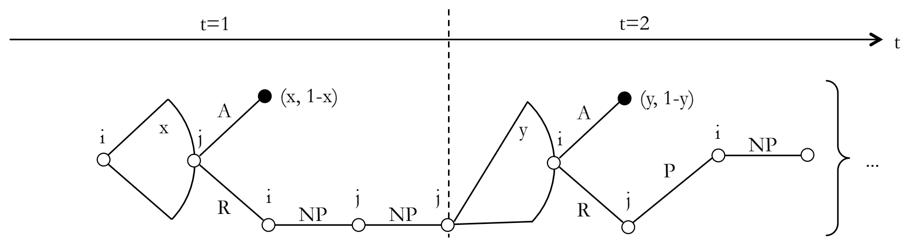

Figure 3.3: The Structure of *Regime j*

To illustrate, consider the case where i is in a position with maximal bargaining power. He will have maximal bargaining power when he chooses to always punish j after his offer is rejected, whereas j never punishes i. Call this *Regime i*, or R_i, the structure of which is graphically illustrated in figure 3.2. In *Regime i*, i will then be able to assure himself the payoff derived in (3.2) with $c_{ji} = c_{jj} = 0$, which I will label $x_{ii}^{R_i}$. This leaves j a share of $x_{ij}^{R_i} = 1 - x_{ii}^{R_i}$. More specifically, the shares of players i and j correspond to:

$$(x_{ii}^{R_i}, x_{ij}^{R_i}) = \\ (\frac{1 - \delta_j + \delta_i\delta_j\alpha_i - \delta_i\delta_j c_{ii} + c_{ij}}{1 - \delta_i\delta_j + \delta_i\delta_j\alpha_i + \delta_j\alpha_j}, \frac{\delta_j - \delta_i\delta_j + \delta_j\alpha_j + \delta_i\delta_j c_{ii} - c_{ij}}{1 - \delta_i\delta_j + \delta_i\delta_j\alpha_i + \delta_j\alpha_j}) \tag{3.4}$$

In contrast, *Regime j*, the structure of which is graphically illustrated in figure 3.3, consists of j's punishment whenever his offers are rejected, and i's never punishing. Note however, that in *Regime j* player i is still the first to make an offer.

The resulting equilibrium division in this regime is:

$$(x_{ii}^{R_j}, x_{ij}^{R_j}) = \\ (\frac{1-\delta_j+\delta_i\delta_j\alpha_i-\delta_j c_j+\delta_j c_{jj}}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_i+\delta_j\alpha_j}, \frac{\delta_j-\delta_i\delta_j+\delta_j\alpha_j-\delta_j(c_{ji}-c_{jj})}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_i+\delta_j\alpha_j}) \tag{3.5}$$

Now assume that i believes that if he left the rejection of his offer x^{R_i} by j unpunished, the equilibrium of the alternating offers game without punishment would evolve.[8] His share in this case is x_{ji}^0, which corresponds to x_{ji}^P for $c_{jj} = c_{ji} = 0$. If i chooses to punish a rejection, he would incur costs of c_{ii}, and the game would be delayed by one period. In the next period, the best player i can expect to receive is $x_{ji}^{R_i}$. It is now rational for i to indeed punish j if the costs from doing so, c_{ii}, plus the future income $x_{ji}^{R_i}$ exceed the payoff from avoiding costs today and receiving an income of x_{ji}^0 tomorrow. Formally:

$$-c_{ii} + \delta_i x_{ji}^{R_i} > 0 + \delta_i x_{ji}^0 \tag{3.6}$$

It follows that, if (3.6) holds, i will make an offer according to (3.2), which will immediately be accepted by j. Hence, if punishment is rational for i, then R_i constitutes a subgame perfect equilibrium supporting the partition $(x_{ii}^{R_i}, x_{ij}^{R_i})$.[9]

Result 2 *Ceteris paribus*, players will be willing to incur larger costs of punishment, the more they "dislike" their opponents, i.e. the smaller the other-regarding component in their utility functions.

Proof. Inequality (3.6) holds for sufficiently small costs of punishment, more precisely for values of $c_{ii} < \delta_i(x_{ji}^{R_i} - x_{ji}^0)$. Because $\frac{\partial x_{ji}^{R_i}}{\partial \alpha_i} < 0$ and

[8] To clarify, if only i has the option to punish, but chooses not to do so, then this scenario corresponds to the alternating offers game without punishment.

[9] For the sake of precision: The strategies supporting R_i are as follows. Player i offers $x_{ij}^{R_i}$ and accepts all counter-offers which yield him $x \geq (1 - x_{ji}^{R_i})$. All counter-offers $x < (1 - x_{ji}^{R_i})$ will be rejected by i. All rejections by j are punished. Player j offers $x_{ji}^{R_i}$, accepts all $x \geq x_{ij}^{R_i}$ and rejects all $x < x_{ij}^{R_i}$. Deviations from this profile will never be profitable for j. In case he would deviate, he would have to incur the punishment without expecting any larger future payoff. Player i finds it profitable to invest in punishment in order to implement his regime R_i if (3.6) holds.

$\frac{\partial x_{ji}^0}{\partial \alpha_i} > 0$ (note that $x_{ji}^0 = 1 - x_{jj}^0$ and equals the analysis for x_{ii} from section before), the right-hand side of this term will diminish for increasing values of α_i and increase for decreasing values. Hence, players would be willing to incur larger costs of punishment, the more they "dislike" the other, i.e. the smaller α_i. □

An example may be instructive: Consider the case that players are indefinitely patient, i.e. $\delta_i = \delta_j = 1$. Substituting and rearranging the relevant terms in (3.6) yields the following condition rendering punishment rational:[10]

$$\frac{c_{ii}}{c_{ij}} < \frac{1}{1 + \alpha_i + \alpha_j} \tag{3.7}$$

Hence, what matters for rational punishment decisions is not the costs of punishment per se, but rather their effectiveness, as long as (3.6) holds. That is, the cheaper it is for either player to reduce the payoff of the other player by a given amount, the more likely the punishment option will be chosen. As can be seen from (3.7), the threshold level of the corresponding cost ratio decreases in the other-regarding components of the players' preferences.

4. *"True Conflict": Inefficient Equilibria*

It is worth noting that the equilibria derived in the previous section do not implement inefficiencies. Because player j has no incentive to reject an offer, outcomes will still be efficient. More specifically, if (3.6) holds, any offered partition $\overline{x} \in [x_{ii}^{R_j}, x_{ii}^{R_i}]$ can be supported as a subgame perfect equilibrium without delay.

Still, inefficient equilibria may result if players are able to punish each other. The threat of "loosing one's regime", and the loss of monetary payoffs this implies, may induce players to punish their opponent. To see how inefficient equilibria arise, consider the case that both players punish each other over N periods, before coming to a compromised agreement x^C. For i, following the path of mutual punishment is more profitable

[10] I derive this inequality from substituting the terms $x_{ji}^{R_i}$ and x_{ji}^0 in (3.6) by the specific values and solving for the relative costs. These are then given as $\frac{c_{ii}}{c_{ij}} < \frac{\delta_i^2 \delta_j}{1 - \delta_i \delta_j (1 - \alpha_j) + \delta_i (1 + \alpha_i)}$.

than immediately giving in to j's regime R_j iff:

$$\sum_{t=0}^{N-1} \delta_i^t(-c_{ii} - c_{ji}) + \delta_i^N x_{ii}^C \geq x_{ii}^{R_j}, \tag{4.1}$$

or

$$x_{ii}^C \geq (x_{ii}^{R_j} + \frac{1-\delta_i^N}{1-\delta_i}(c_{ii} + c_{ji}))\delta_i^{-N}. \tag{4.2}$$

Likewise, it is rational for j to stick to the punishment if his share $(1 - x_{ii}^C)$ is larger or equal than the costs of punishment incurred over N periods, plus the payoff he would receive in i's regime in period $N+1$. Formally, this will be the case for values of x_{ii}^C satisfying the following condition:

$$x_{ii}^C \leq 1 - (x_{ij}^{R_i} + \frac{1-\delta_j^N}{1-\delta_j}(c_{ij} + c_{jj})\delta_j^{-N}). \tag{4.3}$$

Together, conditions (4.4) and (4.3) open a range of possible compromise agreements, x_{ii}^C, for which it is rational for both players to punish each other for N periods. This range is defined as follows:

$$x_{ii}^C \in [(x_{ii}^{R_j} + \frac{1-\delta_i^N}{1-\delta_i}(c_{ii}+c_{ji}))\delta_i^{-N}, 1-(x_{ij}^{R_i} + \frac{1-\delta_j^N}{1-\delta_j}(c_{ij}+c_{jj})\delta_j^{-N})] \tag{4.4}$$

Note that the boundaries of x_{ii}^C depend on $x_{ii}^{R_j}$ and $x_{ij}^{R_i}$, which in turn depend on the other-regarding utilities. More specifically, the following result can easily be seen from equations (3.2) to (3.5).

Result 3 *Ceteris paribus*, the more positively other-regarding the first-moving (second-moving) player is, i.e. the larger α_i (α_j), the smaller will be the range of equilibria for which mutual punishment is rational.

Proof. As α_i (α_j) increases, the lower (upper) bound of x_{ii}^C increases (decreases). Hence, the range of equilibria for which mutual punishment is rational is reduced. □

Result 4 *Ceteris paribus*, the more positively other-regarding the first-moving (second-moving) player is, i.e. the larger α_i (α_j), the fewer will the periods of disagreement (and punishment) be. The degree of inefficiency will be reduced given (positive) other-regarding preferences.

Proof. Obvious from (4.1) and (4.4). □

5. *Conclusion*

In the above it was shown that other-regarding preferences may complicate the finding of a compromise in a bargaining situation. The implication of the analysis is that negative other-regarding preferences have to be reduced to facilitate the finding of an agreement. One way in which this can be done is illustrated in Kellen and Maoz (2012), who analyze the impact of a shared "Peacecamp Identity" in a track two workshop between Palestinians and Israelis. Accordingly, such a "Peacecamp Identity" can be built if the negotiators are physically separated from the parties they represent, and bargain behind closed doors. The idea is that negotiators will come to respect and possibly even sympathize with their opponents.[11]

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[11]It should be noted, however, that the authors conclude that this superordinate identity made hinder the effectiveness of the conflict resolution mechanism by rendering the delegates less representative of their respective groups.

Appendix

Assume player i makes offers in any odd-numbered period. The maximum amount x he may be able to set through can be derived via backward induction. If the game carried on to period $t = 3$ then the maximum amount would yield him a level of utility of

$$u_i^{t=3} = x + \alpha_i(1-x) - c_{ii} - c_{ji}.$$

He would be indifferent between receiving x in period $t = 3$ and receiving $\delta_i u_i^{t=3}$ in the preceding period $t = 2$, in which player j makes an offer. Knowing this, rational j would have to offer i exactly the amount which makes i indifferent between his payoffs in period $t = 2$ and period $t = 3$. This would leave j with a level of utility of

$$u_j^{t=2} = 1 - \delta_i(x + \alpha_i(1-x) - c_{ii} - c_{ji}) + \alpha_j x - c_{ij} - c_{jj}$$

In period $t = 1$, in which i makes an offer, this would leave him with a utility of $\delta_j u_j^{t=2}$. Hence, a rational player i would have to offer him exactly this amount, which leaves him with

$$u_i^{t=1} = 1 - \delta_j(1 - \delta_i(x + \alpha_i(1-x) - c_{ii} - c_{ji}) + \alpha_j x - c_{ij} - c_{jj}) - c_{ii} - c_{ji}$$

Since the game is stationary, the decision problem i faces is the same in all odd-numbered periods. Put differently, the maximum levels of utility in period $t = 1$ and $t = 3$ will be the same. Hence

$$x + \alpha_i(1-x) - c_{ii} - c_{ji} =$$
$$1 - \delta_j(1 - \delta_i(x + \alpha_i(1-x) - c_{ii} - c_{ji}) + \alpha_j x - c_{ij} - c_{jj}) + \alpha_i(1-x) - c_{ii} - c_{ji}.$$

Rearranging:

$$x = 1 - \delta_j + \delta_i\delta_j x + \delta_i\delta_j\alpha_i(1-x) - \delta_i\delta_j(c_{ii} + c_{ji}) - \delta_j\alpha_j x + \delta_j(c_{ij} + c_{jj})$$

$$\Rightarrow x(1 - \delta_i\delta_j + \delta_i\delta_j\alpha_i + \delta_j\alpha_j) = 1 - \delta_j + \delta_i\delta_j\alpha_i - \delta_i\delta_j(c_{ii} + c_{ji}) + \delta_j(c_{ij} + c_{jj})$$

$$\Rightarrow x = \frac{1 - \delta_j + \delta_i\delta_j\alpha_i - \delta_i\delta_j(c_{ii} + c_{ji}) + \delta_j(c_{ij} + c_{jj})}{(1 - \delta_i\delta_j + \delta_i\delta_j\alpha_i + \delta_j\alpha_j)}$$

Alternating Offers Model	First Mover	Share i	Share j
Standard	i	$x_{ii}^{AO} = \frac{1-\delta_j}{1-\delta_i\delta_j}$	$x_{ij}^{AO} = \frac{\delta_j(1-\delta_j)}{1-\delta_i\delta_j}$
	j	$x_{ji}^{AO} = \frac{\delta_i(1-\delta_j)}{1-\delta_i\delta_j}$	$x_{jj}^{AO} = \frac{1-\delta_i}{1-\delta_i\delta_j}$
Other-Regarding Preferences	i	$x_{ii}^{0} = \frac{1-\delta_j-\delta_i\delta_j\alpha_i}{1-\delta_i\delta_j(1+\alpha_i)+\delta_j\alpha_j}$	$x_{ij}^{0} = \frac{\delta_j(1+\alpha_j)-\delta_i\delta_j}{1-\delta_i\delta_j(1+\alpha_i)+\delta_j\alpha_j}$
	j	$x_{ji}^{0} = \frac{\delta_i(1+\alpha_i)-\delta_i\delta_j}{1-\delta_i\delta_j(1+\alpha_j)+\delta_j\alpha_i}$	$x_{jj}^{0} = \frac{1-\delta_i-\delta_i\delta_j\alpha_j}{1-\delta_i\delta_j(1+\alpha_j)+\delta_j\alpha_i}$
Punishment	i	$x_{ii}^{P} = \frac{1-\delta_j+\delta_i\delta_j\alpha_i-\delta_i\delta_j(c_{ii}+c_{ji})+(c_{ij}+c_{jj})}{(1-\delta_i\delta_j+\delta_i\delta_j\alpha_i+\delta_j\alpha_j)}$	$x_{ij}^{P} = \frac{\delta_j-\delta_i\delta_j+\delta_j\alpha_j+\delta_i\delta_j(c_{ii}+c_{ji})-(c_{ij}+c_{jj})}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_i+\delta_j\alpha_j}$
	j	$x_{ji}^{P} = \frac{\delta_i-\delta_i\delta_j+\delta_j\alpha_i+\delta_i\delta_j(c_{jj}+c_{ij})-(c_{ji}+c_{ii})}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_j+\delta_j\alpha_i}$	$x_{jj}^{P} = \frac{1-\delta_i+\delta_i\delta_j\alpha_j-\delta_i\delta_j(c_{jj}+c_{ij})-(c_{ji}+c_{ii})}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_j+\delta_j\alpha_i}$

Table 1: Equilibrium Outcomes of Different Versions of the Alternating Offers Model

Regime	First Mover	Share i	Share j
Regime R_i	i	$x_{ii}^{R_i} = \frac{1-\delta_j+\delta_i\delta_j\alpha_i-\delta_i\delta_j c_{ii}+c_{ij}}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_i+\delta_j\alpha_j}$	$x_{ij}^{R_i} = \frac{\delta_j-\delta_i\delta_j+\delta_j\alpha_j+\delta_i\delta_j c_{ii}-c_{ij}}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_i+\delta_j\alpha_j}$
	j	$x_{ji}^{R_i} = \frac{\delta_i-\delta_i\delta_j+\delta_i\alpha_i-\delta_i(c_{ij}-c_{ii})}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_j+\delta_i\alpha_i}$	$x_{jj}^{R_i} = \frac{1-\delta_i+\delta_i\delta_j\alpha_j-\delta_i c_i+\delta_i c_{ii}}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_j+\delta_j\alpha_j}$
Regime R_j	i	$x_{ii}^{R_j} = \frac{1-\delta_j+\delta_i\delta_j\alpha_i-\delta_j c_j+\delta_j c_{jj}}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_i+\delta_j\alpha_j}$	$x_{ij}^{R_j} = \frac{\delta_j-\delta_i\delta_j+\delta_j\alpha_j-\delta_j(c_{ji}-c_{jj})}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_i+\delta_j\alpha_j}$
	j	$x_{ji}^{R_j} = \frac{\delta_i-\delta_i\delta_j+\delta_i\alpha_i+\delta_i\delta_j c_{jj}-c_{ji}}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_j+\delta_i\alpha_i}$	$x_{jj}^{R_j} = \frac{1-\delta_i+\delta_i\delta_j\alpha_j-\delta_i\delta_j c_{jj}+c_{ji}}{1-\delta_i\delta_j+\delta_i\delta_j\alpha_j+\delta_i\alpha_i}$

Table 2: Equilibrium Outcomes of Different Punishment Regimes