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Offshoring and Sequential Production Chains:
A General Equilibrium Analysis

Philipp Harms*, Jaewon Jung† and Oliver Lorz‡

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Abstract

We present a two-region general equilibrium model in which firms exploit international wage differences by offshoring parts of the production process. Firms have to take into account that production steps follow a strict sequence and that transporting intermediate goods across borders is costly. We analyze how a change in transport costs and various properties of the production process affect the volume of offshoring, accounting for the general equilibrium effects of firms’ decisions. Interestingly, the influence of declining transport costs on the range of tasks offshored per firm may differ from the effect on the number of firms engaged in offshoring.

Keywords: Offshoring, sequential production, global production chain, task trade.

JEL Classification: D24, F10, F23, L23.

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1 Introduction

Over the past years a lot of attention has been devoted to the determinants and consequences of the “second unbundling” in international trade (Baldwin 2006) – i.e., the international fragmentation of production. Many recent analyses of this phenomenon are based on a specific idea of the production process, according to which production can be interpreted as a set of “tasks” or “production steps”. The decision to offshore a certain task depends on relative factor prices and productivity levels as well as on offshoring costs for that particular task (including additional monitoring and communication costs resulting from foreign production). Individual tasks can be ordered with respect to the cost advantage of performing them abroad, and there is a unique “cutoff task” that defines the extent of offshoring.

In contrast to this approach, some recent contributions explicitly consider the fact that, in many cases, production processes are sequential – i.e., individual steps follow a predetermined sequence that cannot be modified at will. With sequential production, offshoring implies unfinished intermediate goods to be transported back and forth between countries – a phenomenon whose empirical relevance is documented, e.g., by Haddad (2007), Ando and Kimura (2005), as well as Athukorala and Yamachita (2006): these studies describe the behavior of Japanese multinationals who ship high-technology core materials to their affiliates in developing East Asia, where they produce basic parts and components; the basic parts and components are then sent back to Japan (or to other high-skill abundant countries) for quality control and/or further processing.

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1 A non-exhaustive list of important contributions to this literature includes Jones and Kierzkowski (1990), Feenstra and Hanson (1996), Kohler (2004), and Grossman and Rossi-Hansberg (2008).

At a more aggregate level, recent balance of payments data document an increase of “manufacturing services on physical inputs owned by others”. Figure 1 shows the volume of manufacturing services exported by a set of Asian as well as Central and Eastern European countries. It illustrates the absolute volume as well as the growth of manufacturing services exports.

The existence of sequential production would not necessarily change our view on offshoring if the relative costs of performing individual tasks abroad happened to monotonically increase or decrease along the production process. However, it is quite unlikely

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3. This novel category within the services trade part of the current account was introduced in the context of the sixth revision of the Balance of Payments Manual (BPM6). Since time series that comply with BPM6 only go back to 2005, we do not have data on this item for previous years.

4. The Asian countries are Bangladesh, China, India, Indonesia, Korea, Malaysia. The Central and Eastern European countries are Bulgaria, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovak Republic, Slovenia, Turkey. Country-years for which no data were available entered the sum with a zero.
to meet this constellation in practice. More plausible appears to be a setting in which potential offshoring destinations have a cost advantage for some particular segments of the production process whereas preceding and subsequent segments can be performed at lower costs in the domestic economy. If such a situation is combined with costs of shipping intermediate goods across borders, firms may be reluctant to offshore certain steps even if – considered in isolation – these could be performed at much lower costs abroad. The reason is that the domestic country may have a cost advantage with respect to adjacent steps, and the costs of shifting back and forth intermediate goods may more than eat up potential cost savings from fragmenting the production process. Such a constellation has important implications for observed offshoring patterns. For example, it may explain why – despite the large international discrepancies in factor prices – certain production processes are less fragmented internationally than what one might expect. At the same time, such a setup may generate substantial shifts in the total volume of offshoring as a reaction to rather moderate changes of the environment. And finally, it may give rise to a non-monotonic relationship between transport costs and the volume of offshoring.

Baldwin and Venables (2013) as well as Harms et al. (2012) have shown how such insights regarding the offshoring decision of individual firms can be obtained from partial equilibrium models in which factor prices are exogenous. However, to arrive at conclusions about the entire economy, we need to account for the influence of firms’ offshoring decisions on factor prices at home and abroad, and we have to consider the repercussions of induced factor price changes on firms’ optimal behavior – i.e., we need to model offshoring in a general equilibrium framework. This is what the current paper does.

Baldwin and Venables (2013) model firms’ offshoring decisions for different configurations of production processes – “snakes” and “spiders” – with spiders reflecting a situation in which different intermediate inputs may be simultaneously produced in different countries to be eventually assembled at a central location, and snakes capturing the notion of sequentiality outlined in the introduction. Using these alternative frameworks, they analyze the consequences of exogenous variations in production costs and offshoring costs, including costs of shipping intermediate goods across borders. Highlighting the “tension between comparative costs creating the incentive to unbundle, and co-location or agglomeration forces binding stages of production together” (Baldwin and Venables 2013: 246),
which is also characteristic for our setup, they show that a decrease in shipping costs may result in an **overshooting** of the overall offshoring pattern: production stages may first be shifted abroad to take advantage of co-location and then return to the domestic country as shipping costs further decrease. In Harms et al. (2012), relative costs fluctuate symmetrically along the production process, allowing a deeper analysis of the technological factors that influence the offshoring decision. In addition to shipping costs, Harms et al. (2012) consider communication and supervision costs caused by offshoring activities as well as the variability of these costs across tasks, or the total length of the production chain.

The current paper starts from a similar symmetric specification of a production- “snake” and places this in a general equilibrium context. We present a two-region (North-South) model in which firms whose production is entirely **domestic** may co-exist with **multinational** firms who decide on the international distribution of production. By allowing for firms with different “production modes”, we avoid the drastic adjustment that characterized the model of Harms et al. (2012): instead of modeling an economy in which a representative firm possibly jumps from purely domestic production to a large volume of offshoring, we describe the adjustment as taking place both at the extensive and the intensive margin – i.e., as a reaction to changing parameters, both the number of (small) firms who engage in offshoring and the volume of offshoring chosen by these firms may change. Production is based on a rigid sequence of individual steps, and the foreign cost advantage evolves in a non-monotonic fashion along the production chain: some steps are cheaper to perform in the South, some are cheaper to perform in the North, and so on. Finally, every step requires the presence of the unfinished intermediate good, and shifting intermediate goods across borders is associated with transport costs. Wages and prices in both economies are endogenous, and the increasing demand for labor that is generated by accelerating North-South offshoring may eventually result in wage increases that make offshoring less attractive. Using this framework, we explore how changes in transport costs affect the volume of offshoring at the intensive and extensive margin as well as factor prices in the North and the South. Numerical simulations corroborate our comparative-static results and provide further insights into the effects of changing factor endowments, relative productivities, and properties of the production process.

In another related approach, Costinot et al. (2012, 2013) use a general equilibrium...
model with multiple countries to analyze how a countries’ productivity – as reflected by its propensity to commit mistakes – determines the stages of production it attracts. These authors also emphasize the concept of sequentiality, i.e., the idea that the order in which tasks have to be performed is exogenously determined. Moreover, the transport costs in our model play a role that is somewhat similar to the “coordination costs” in Costinot et al. (2013: 111), with a decrease in these costs inducing a non-monotonic adjustment of the overall volume of offshoring. However, we deviate from the assumption in Costinot et al. (2012, 2013), which stipulates that countries can be ordered by their relative productivities. Conversely, we model a two-region world in which the relative cost advantage of the North fluctuates non-monotonically along the production chain. Using this framework we analyze how specific properties of industry-specific production processes affect the volume of offshoring and relative factor prices across countries.

The rest of the paper is organized as follows: in section 2, we outline the structure of our model. Section 3 discusses the properties of the equilibrium and derives comparative static results. In section 4, we perform a numerical analysis that illustrates how the volume of offshoring is affected by changes in factor endowments, decreasing transport costs, and various properties of the production process. Section 5 provides a summary and some conclusions.

2 The Model

2.1 Preferences

There are two regions, North and South, with an asterisk denoting South-specific variables.\(^5\) Consumers in both regions have Cobb-Douglas preferences over two consumption goods, \(X\) and \(Y\). The \(X\) sector produces a continuum of differentiated varieties, whereas

\(^5\)Each region should be understood as an aggregate composed of possibly numerous countries.
goods from the \( Y \) sector are homogeneous.\(^6\) Household preferences are

\[
U = X^\beta Y^{1-\beta}, \quad 0 < \beta < 1, \quad \text{and}
\]

\[
X = \left[ \int_{i \in N} x(i)^{\rho_x} di \right]^{\frac{1}{\sigma_x}}, \quad 0 < \rho_x < 1. \quad (1)
\]

The index \( i \) denotes individual varieties, \( N \) is the measure of these varieties, and \( \sigma_x = 1/(1-\rho_x) \) is the elasticity of substitution between them. Maximizing utility for a given income level \( I \) yields the following demand system:

\[
x(i) = \left( \frac{P_X}{p(i)} \right)^{\sigma_x} X, \\
P_X = \left[ \int_{i \in N} p(i)^{1-\sigma_x} di \right]^{\frac{1}{1-\sigma_x}}, \quad P_X X = \beta I, \quad \text{and} \quad p_Y Y = (1-\beta)I. \quad (2)
\]

Here, \( P_X \) denotes the ideal price index for the \( X \)-sector and \( p_Y \) denotes the price of the homogenous output in industry \( Y \).

### 2.2 Technologies and Production Modes

Each region is endowed with fixed quantities of labor \( \bar{L} \) (in efficiency units) and a fixed composite factor \( \bar{R} \). We assume that labor can be employed in both sectors, whereas the fixed composite factor, which may be land or a natural resource, is specific to industry \( Y \). Good \( Y \) is produced in both regions by a competitive industry and can be freely traded.

Production of \( Y \) combines the quantities \( R_Y \) and \( L_Y \) according to a CES-technology:

\[
Q_Y = \left[ \alpha_R R_Y^{\sigma_Y} + \alpha_L L_Y^{\psi_Y} \right]^{\frac{1}{\sigma_Y}}, \quad 0 < \rho_Y < 1. \quad (3)
\]

Profit maximization yields the following demand for the two factors of production in the \( Y \)-sector:

\[
R_Y = \alpha_R^{\sigma_Y} \left( \frac{P_Y}{r} \right)^{\sigma_Y} Q_Y \quad \text{and} \quad L_Y = \alpha_L^{\sigma_Y} \left( \frac{P_Y}{w} \right)^{\sigma_Y} Q_Y. \quad (4)
\]

The term \( \sigma_Y = 1/(1-\rho_Y) \) stands for the elasticity of substitution between the two factors \( R_Y \) and \( L_Y \), and \( r \) and \( w \) denote factor prices, respectively. If good \( Y \) is produced, perfect

\(^6\) As we will detail below, offshoring exclusively takes place in the \( X \)-sector. The two-sector framework is used in order to guarantee that foreign workers find an alternative employment if domestic firms do not offshore any tasks.
competition implies that
\[ p_Y = \left[ \alpha_R^{\sigma_Y} r^{1-\sigma_Y} + \alpha_L^{\sigma_Y} w^{1-\sigma_Y} \right]^{-\frac{1}{\sigma_Y}}. \] (5)

We choose good \( Y \) as our numeraire throughout the paper, and from free trade in \( Y \) and product homogeneity we have \( p_Y = p^*_Y = 1 \).

Varieties of good \( X \) can only be produced by firms whose headquarters are located in the North. While we do not explicitly model research and development, this assumption can be rationalized by arguing that only Northern firms are able to develop and use the blueprints necessary for production. Firms in sector \( X \) act under monopolistic competition, and each firm has to spend fixed costs in addition to the variable production costs.\(^7\)

We follow Grossman and Rossi-Hansberg (2008) in modeling the production process of any variety \( x(i) \) as a continuum of tasks, indexed by \( t \) and ranging from 0 to 1. As in Harms et al. (2012) and Baldwin and Venables (2013), these tasks have to be performed following a strict sequence, i.e., they cannot be re-arranged at will. A firm producing a given variety of good \( X \) may choose between different production modes: domestic (\( D \)) and multinational (\( M \)). While domestic firms perform the entire production process in the North, multinational firms can offshore some of the tasks to the South. The goal of our analysis is to determine the number of good-\( X \) producers that exploit the possibility to produce internationally and to derive the amount of offshoring chosen by these firms.

Each task \( t \) involves a given quantity of labor.\(^8\) The labor coefficients \( a_D(t) \) or \( a_M(t) \) denote the efficiency units of labor that are necessary for a domestic (\( D \)) or multinational (\( M \)) firm to perform task \( t \) in the North. For simplicity, we assume that \( a_D(t) = a_D \) and \( a_M(t) = a_M \) for all \( t \), i.e., input coefficients in the North do not differ across tasks.\(^9\) Being a multinational firm may come with a higher labor productivity in the North, i.e., \( a_M \leq a_D \).

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\(^7\)The monopolistic-competition framework with product differentiation allows for the co-existence of firms with different production modes in equilibrium.

\(^8\)Given the linear specification of production in sector \( X \), the existence of sector \( Y \) and the specification of its technology add some convexity to the model. See, e.g., Markusen and Venables (1998) as well as Markusen (2002) for a similar approach.

\(^9\)By contrast to this assumption of fixed labor coefficients, Jung and Mercenier (2014), who analyze skill/technology upgrading effects of globalization, employ a heterogeneous workers framework in which a worker’s productivity is determined endogenously by his own talent and the technology he uses.
Input coefficients $a^*_M(t)$ of performing a task in the South possibly differ from $a_M$, and—more importantly—these coefficients vary across tasks. For example, some tasks benefit more strongly from a better educated workforce or a better production infrastructure in the North, implying a lower labor coefficient in the North than in the South. Other tasks may be less sophisticated such that the effective labor input for these tasks may be lower in the South than in the North. In addition, offshoring firms possibly have to employ additional labor to monitor tasks performed in the South or to communicate with the headquarter in the North, and these monitoring and communication requirements may also vary across tasks. Summing up, there may be tasks which—given wages $w$ and $w^*$ in the North and the South respectively—are cheaper to perform domestically and tasks which are cheaper to perform abroad. This is represented by Figure 2, which juxtaposes the (constant) costs per task $wa_D$ if these tasks are performed by a Northern domestic firm, the (constant) costs $wa_M$ if they are performed domestically by a multinational firm, and the varying costs $w^*a^*_M(t)$ if these tasks are offshored to the South.

Multinational firms have to adjust to the fact that, at given wages, some tasks that are cheaper to perform in the South may be adjacent to tasks for which the North has a cost advantage (and vice versa). In Figure 2, the tasks $t \in [t_1, t_2]$ and $t \in [t_3, t_4]$ would be performed at lower cost if they were offshored to the South by Northern multinational firms. Conversely, all tasks $t \in [0, t_1]$, $t \in [t_2, t_3]$ and $t \in [t_4, 1]$ would be cheaper to perform domestically. To simplify the analysis, we assume that the first and the last task are tied to being performed in the North.

![Figure 2: Costs along the Production Chain](image)
As in Yi (2003, 2010), Barba Navaretti and Venables (2004), or Harms et al. (2012), we furthermore assume that production tasks require the presence of the unfinished intermediate good, and that moving (intermediate) goods between regions is associated with constant transportation costs per unit. More specifically, any crossing of regional borders requires \( T \) units of labor in the sending region. It is for this reason that Northern multinational firms may find it profitable to perform a large part of production at home or to adopt a strategy of *agglomerating* (almost) the entire production process \( t \in [t_1, t_4] \) abroad, rather than paying transport costs each time the unfinished good is crossing borders.

In the rest of this paper, we restrict our attention to a symmetric specification of the \( a_M^*(t) \) curve: \(^{10}\)

\[
a_M^*(t) = A \cos(2n\pi t) + B. \tag{6}
\]

The symmetry engrained in this specification offers a flexible way to capture the non-monotonic evolution of relative costs along the value-added chain while substantially simplifying the analysis: first and foremost, instead of determining separate cutoff values \( t_1, t_2, ..., t_{2n} \), we can exploit the fact that \( t_2 = \frac{1}{n} - t_1, t_3 = \frac{1}{n} + t_1, \) etc. The first cutoff \( t_1 \) thus determines all critical values \( t_i \). Moreover, the individual parameters characterizing the cosine-function have a straightforward economic interpretation: while the shift parameter \( B \) reflects the average labor coefficient in the South, the parameter \( A \), which determines the function’s amplitude, captures the heterogeneity of task-specific input requirements across regions. The variable \( n (n \in \mathbb{N}^+) \) measures the number of “cycles” that \( a_M^*(t) \) completes between \( t = 0 \) and \( t = 1 \). We argue that production processes that are characterized by a higher number of cycles – i.e., a larger \( n \) – are more sophisticated, exhibiting more variability in terms of cost differences. To keep the analysis interesting, we assume that foreign production costs fluctuate around domestic costs more than once \((n \geq 2)\).

Offshoring with positive production volumes in both regions can only occur if each region has a cost advantage for some tasks. Technically, this requires that the \( w^*a_M^*(t) \)

\(^{10}\)This cosine specification is similar to the working paper version of Harms et al. (2012), which, however, did not incorporate general equilibrium effects. Various other specific functional forms, consistent with our assumptions, could be adopted, but with no additional insight or tractability gained.
curve in Figure 2 intersects the $wa_M$ line at least once. Therefore, a necessary condition for international production sharing in our paper is

$$\frac{B - A}{a_M} < \frac{w}{w^*} < \frac{B + A}{a_M}. \quad (7)$$

Due to the symmetric specification of the $a^*_M(t)$ function, we may distinguish three firm-types: domestic firms ($domestic$ $production$, $D$), multinational firms that fragment their production chain ($fragmentation$, $M^f$), or multinational firms that perform most tasks in the South ($production$ $abroad$, $M^a$). Fragmented multinationals offshore all segments that are cheaper to perform in the South, i.e., all tasks between $t_i$ and $t_{i+1}$ ($i = 1, 3, \ldots, 2n - 1$), and produce all other segments at home. Production-abroad firms offshore the entire segment between $t_1$ and $t_{2n}$, and produce only the first segment between 0 and $t_1$ and the last segment between $t_{2n}$ and 1 at home.

### 2.3 Costs and Prices

Marginal costs of firm type $j$ are given by the following expression:

$$C_j = wL_j + w^*L^*_j, \quad (8)$$

for $j = D, M^f, M^a$. The variables $L_j$ and $L^*_j$ stand for the labor input at home and abroad per unit of output of a representative type-$j$ firm, given by

$$L_D = a_D, \quad (9)$$

$$L_{M^f} = 2na_Mt_1 + nT, \quad (10)$$

$$L_{M^a} = 2a_Mt_1 + T, \quad (11)$$

$$L^*_D = 0, \quad (12)$$

$$L^*_{M^f} = n \int_{t_1}^{1/t_1} (A \cos(2\pi nt) + B) \, dt + nT$$

$$= B [1 - 2nt_1] - \frac{A}{\pi} \sin(2\pi nt_1) + nT, \quad \text{and} \quad (13)$$

$$L^*_{M^a} = \int_0^1 (A \cos(2\pi nt) + B) \, dt - 2\int_0^{t_1} (A \cos(2\pi nt) + B) \, dt + T$$

$$= B [1 - 2t_1] - \frac{A}{n\pi} \sin(2\pi nt_1) + T. \quad (14)$$
Note that, with \( n \) cycles, fragmented firms \((M^f)\) have \( 2n + 1 \) sequential production stages, and labor demand for transport of \( n \) times \( T \) in each region, while production-abroad firms \((M^a)\) have three sequential production stages and labor demand for transport of \( T \) in each region.

We assume that exporting final \( X \)-goods to the South is associated with iceberg trade costs \( \tau > 1 \) per unit. Using this information as well as (2), we can write the demand faced by a representative firm of type \( j \) as

\[
x^d_j = \left( \frac{P_X}{p_j} \right)^{\sigma_x} X \quad \text{and} \quad x^{ds}_j = \tau \left( \frac{P^*_X}{p_j \tau} \right)^{\sigma_x} X^*.
\]  

The price indices for the \( X \) sector in the North and the South are given by

\[
P_X = \left[ \sum_j N_j p_j^{1-\sigma_x} \right]^{\frac{1}{1-\sigma_x}} \quad \text{and} \quad P^*_X = \left[ \sum_j N_j (p_j \tau)^{1-\sigma_x} \right]^{\frac{1}{1-\sigma_x}},
\]  

where \( N_j \) stands for the mass of firms of each firm-type. If type-\( j \) firms in sector \( X \) produce a strictly positive quantity \((x_j > 0)\), they charge a constant mark-up rate over their marginal costs:

\[
p_j = \frac{\sigma_x}{\sigma_x - 1} C_j.
\]  

We assume that all active firms have to incur fixed costs that take the form of unsold final goods, i.e., they amount to a multiple of marginal costs \( F_j C_j \), with \( F_j > 0 \), and we assume \( F_D < F_{M^f} < F_{M^a} \). Free entry ensures zero profits:

\[
\frac{1}{\sigma_x} p_j x_j \leq C_j F_j,
\]  

where \( N_j > 0 \) if the respective condition holds with equality, and \( N_j = 0 \) otherwise. With positive production, we can combine (17) with (18) to derive production of a firm of type \( j \) in equilibrium:

\[
x_j = (\sigma_x - 1) F_j.
\]

\(^{11}\)As is common in the literature, we assume that the fixed organizational and set-up costs are higher abroad than domestically. Consequently, they increase in the range of activities performed abroad, implying \( F_{M^a} > F_{M^f} \).
3 Equilibrium

3.1 Definition

An equilibrium is defined by an optimal cutoff value $\tilde{t}_1$, a vector of wages as well as prices and quantities in the $X$ and $Y$ sectors, and an industrial structure $(N_D, N_{MF}, N_{M^*})$ such that

(i) firms of a given type in the $X$-sector set profit-maximizing prices,

(ii) no firm has an incentive to change its production mode,

(iii) multinational firms of a given type $(M^f, M^a)$ choose the optimal pattern of offshoring, i.e., the optimal cutoff value $\tilde{t}_1$,

(iv) free entry results in zero-profits of all active firms in equilibrium,

(v) in the $Y$-sector, factor prices reflect their marginal products,

(vi) goods and factor markets clear.

3.2 Cutoff Task

The optimal cutoff task $\tilde{t}_1$ for firms who engage in offshoring is implicitly determined by the following condition:\footnote{Recall that, given our symmetric cosine specification, all other cutoffs also satisfy this condition.}

$$ wa_M = w^*a_M^*(\tilde{t}_1) . $$

Using (6) and defining $\omega = w/w^*$, we can obtain $\tilde{t}_1$ as

$$ \tilde{t}_1 = \min \left[ \frac{1}{2\pi n} \arccos \left( \frac{a_M \omega - B}{A} \right) , \frac{1}{2n} \right] , $$

where the condition that $\tilde{t}_1 \leq 1/2n$ is due to the cosine function reaching its minimum at $\pi$. The range of offshored tasks thus depends on the domestic labor coefficient $a_M$, the average foreign coefficient $B$, and on the wage $\omega$ in the North relative to the South. An increase in $\omega$ ceteris paribus lowers $\tilde{t}_1$, i.e.,

$$ \frac{\partial \tilde{t}_1}{\partial \omega} = - \frac{a_M}{2\pi n A \sqrt{1 - \left( \frac{a_M \omega - B}{A} \right)^2} } < 0 . $$

The economic intuition behind this result is straightforward: as the domestic wage relative to the foreign wage increases, firms have an incentive to perform a smaller share of the...
production process at home and a larger share abroad. In a similar fashion, we may
determine the influence of the variables determining average costs, $a_M$ and $B$, as well as
the heterogeneity parameters $A$ and $n$.

3.3 Market Clearing

Labor market equilibrium requires

$$L = L_Y + \sum_j N_j L_j (x_j + F_j) \quad \text{and} \quad L^* = L_Y^* + \sum_j N_j L_j^* (x_j + F_j) ,$$

with $j \in \{D, M^f, M^a\}$ and with $L_j(x_j + F_j)$ as the fixed and variable labor input required
at home by a firm of type $j$ to produce $x_j$ units of output.

For the market of the fixed composite factor to be in equilibrium we need

$$\bar{\bar{R}} = R_Y \quad \text{and} \quad \bar{R}^* = R_Y^* .$$

For the final-goods markets, the equilibrium conditions are

$$x_j = x_j^d + x_j^{d*} \quad \text{as well as} \quad Q_Y + Q_Y^* = Y + Y^* .$$

Finally, incomes depend on factor prices and (fixed) factor endowments:

$$I = r\bar{\bar{R}} + w\bar{\bar{L}} \quad \text{and} \quad I^* = r^*\bar{\bar{R}}^* + w^*\bar{\bar{L}}^* .$$

3.4 Production Regimes

In what follows, we distinguish between equilibria in which all $X$-sector firms have the same
production mode and equilibria in which firms with different production modes co-exist.
We will refer to these constellations as production regimes: a pure domestic production
regime is characterized by $N_{M^f} = N_{M^a} = 0$ and $N_D > 0$, a mixed domestic/fragmented
regime is characterized by $N_{M^a} = 0$ and $N_D > 0, N_{M^f} > 0$, etc.\footnote{Note that mixed regimes with the coexistence of different production modes and thus different marginal costs would never arise in a perfect competition setting.}

In a regime in which different production modes $j$ and $k$ co-exist, equation (19) implies
$x_j/x_k = F_j/F_k$. Since it follows from (15) and (17) that $x_j/x_k = (p_j/p_k)^{-\sigma}$ and $p_j/p_k = C_j/C_k$, we obtain the condition

$$C_k = \left( \frac{F_k}{F_j} \right)^{-1/\sigma} C_j .$$
For further reference we set $F_D = 1$ and define

$$\varphi_{Mf} = (F_{Mf})^{-1/\sigma_x} \quad \text{and} \quad \varphi_{Ma} = (F_{Ma})^{-1/\sigma_x} \quad (27)$$

as the fixed cost advantage of producing domestically compared to fragmented production or production abroad, respectively. Since $F_D < F_{Mf} < F_{Ma}$, we have $\varphi_{Ma} < \varphi_{Mf} < 1$.

From (8) to (14), and combining (6) and (20) we get

$$\frac{C_D}{w} = a_D, \quad \frac{C_{Mf}}{w \varphi_{Mf}} = \frac{1}{\varphi_{Mf}} \left(2na_M \tilde{t}_1 + nT + \frac{a_M \left[ B \left(1 - 2n \tilde{t}_1 \right) - \frac{A}{n^2} \sin \left(2\pi n \tilde{t}_1 \right) + n \right]}{A \cos(2\pi n \tilde{t}_1) + B} \right), \quad \text{and} \quad \frac{C_{Ma}}{w \varphi_{Ma}} = \frac{1}{\varphi_{Ma}} \left(2a_M \tilde{t}_1 + T + \frac{a_M \left[ B \left(1 - 2\tilde{t}_1 \right) - \frac{A}{n^2} \sin \left(2\pi n \tilde{t}_1 \right) + 1 \right]}{A \cos(2\pi n \tilde{t}_1) + B} \right). \quad (28, 29)$$

Figure 3 depicts these “marginal cost curves” as a function of the optimal cutoff value \(\tilde{t}_1\). From taking the derivative, it is straightforward to show that $\frac{C_{Mf}}{w \varphi_{Mf}}$ and $\frac{C_{Ma}}{w \varphi_{Ma}}$ are both increasing in \(\tilde{t}_1\). Furthermore, the line $\frac{C_{Ma}}{w \varphi_{Ma}}$ is steeper than $\frac{C_{Mf}}{w \varphi_{Mf}}$ in the plausible case of $L_{Mf}^* < L_{Ma}^*$, and $\frac{C_{Mf}}{w \varphi_{Mf}} > \frac{C_{Ma}}{w \varphi_{Ma}}$ for $\tilde{t}_1 = 0$ as long as $n$ is sufficiently large. In Figure 3, we assume that these conditions are satisfied.

\[\footnote{We obtain}
\frac{dC_{Mf}}{dt_1} = \frac{L_{Mf}^* 2 \pi n a_M A \sin (2 \pi n \tilde{t}_1)}{\varphi_{Mf} \left[A \cos(2\pi n \tilde{t}_1) + B\right]^2} \quad \text{and} \quad \frac{dC_{Ma}}{dt_1} = \frac{L_{Ma}^* 2 \pi n a_M A \sin (2 \pi n \tilde{t}_1)}{\varphi_{Ma} \left[A \cos(2\pi n \tilde{t}_1) + B\right]^2}.
\]
A strictly positive number of firms choosing production mode $j$—i.e. $N_j > 0$—is compatible with equilibrium if $C_j/\varphi_j \leq C_k/\varphi_k$ for every other production mode $k$. We can thus characterize production regimes by a set of “complementary slackness” conditions:

$$\left(\frac{C_j}{\varphi_j} - \frac{C_k}{\varphi_k}\right) N_j \leq 0, \quad N_k \geq 0, \quad j, k = D, M^f, M^a, \quad k \neq j. \quad (31)$$

As illustrated by Figure 3, a regime with only production-abroad firms ($N_{M^a} > 0, N_D = N_{M^f} = 0$) exists if $\bar{t}_1 < \bar{t}_1^A$, a regime with fragmented production and production abroad ($N_{M^f} > 0, N_{M^a} > 0, N_D = 0$) exists if $\bar{t}_1 = \bar{t}_1^A$, etc. For the three production modes to co-exist, the three curves in Figure 3 would have to intersect in one point. We consider this to be an extremely unlikely outcome, and in what follows we focus on the analysis of equilibria with one or two production modes.

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15We are talking about the “number” of firms knowing that, with a continuum of intermediate products, the term “mass” would be more appropriate, though less intuitive.

16Recall that Figure 3 depicts an example. Depending on parameter values, the existence and ordering of points of intersection may differ from the one depicted.
3.5 Model Solution

Let us take stock: for a given relative wage $\omega$, equation (21) defines the optimal cutoff value $\tilde{\tau}_1$ for firms that engage in international production. This cutoff value can be fed into the labor demand equations (9)–(14) and the “marginal cost curves” (28)–(30). A set of complementary slackness conditions (31) determines which production modes co-exist in equilibrium.

To determine the remaining endogenous variables, we combine the demand and price functions (15) and (16) with the income definitions (25) as well as the “supply function” (19). This yields for $N_j > 0$

$$N_j = \frac{\beta \omega (\bar{L} + \bar{R} \frac{\bar{L}}{\bar{R}}) + \beta (\bar{L}^* + \bar{R}^* \frac{\bar{L}^*}{\bar{R}^*})}{\sigma_x F_j (\omega L_j + L_j^*)} - N_k \left( \frac{\varphi_k}{\varphi_j} \right)^{1-\sigma_x}. \quad (32)$$

Note that $N_k = 0$ is an equilibrium in which all firms choose production mode $j$. It follows from (32) that $N_j$ depends on the relative wage $\omega$ as well as factor price ratios $(r/w)$ and $(r^*/w^*)$. The latter can be derived by combining the labor market equilibrium conditions (22) with (4) and (18), i.e., the conditions characterizing optimal factor demand in the $Y$-sector. This yields

$$\bar{L} - \sigma_x (N_k L_k F_k + N_j L_j F_j) = \left( \frac{\alpha_L}{\alpha_R} \right)^{\sigma_Y} \left( \frac{\bar{r}}{\bar{w}} \right)^{\sigma_Y} \bar{R} \quad \text{and} \quad (33)$$

$$\bar{L}^* - \sigma_x (N_k L_k^* F_k + N_j L_j^* F_j) = \left( \frac{\alpha_L}{\alpha_R} \right)^{\sigma_Y} \left( \frac{\bar{r}^*}{\bar{w}^*} \right)^{\sigma_Y} \bar{R}^*. \quad (34)$$

Finally, the relative wage $\omega$ can be determined by combining (5) with the numeraire assumption $p_Y = p_Y^* = 1$:

$$\omega = \left[ \frac{\alpha_R^{\sigma_Y} \left( \frac{r^*}{w^*} \right)^{1-\sigma_Y} + \alpha_L^{\sigma_Y}}{\alpha_R^{\sigma_Y} \left( \frac{r}{w} \right)^{1-\sigma_Y} + \alpha_L^{\sigma_Y}} \right]. \quad (35)$$

In equilibrium, the relative wage implied by (35) has to coincide with the value of $\omega$ on which the threshold task $\tilde{\tau}_1$ was based.

3.6 Comparative Static Analysis

In this section, we analyze how an exogenous decrease in border-crossing costs ($T$) influences the relative wage and the intensive/extensive margin of offshoring. In a mixed
regime with two production modes, we may describe the equilibrium with the help of two conditions. The “free entry” condition $C_j/\varphi_j = C_k/\varphi_k$ pins down $\tilde{t}_1$. This pattern is represented by the horizontal $FE$ line in Figure 4. The “optimal cutoff” condition (21), which establishes a negative relationship between $\omega$ and $\tilde{t}_1$, is reflected by the downward-sloping $OC$ curve in Figure 4. An equilibrium is characterized by the intersection of the $FE$ line and the $OC$ curve. In this case, the relative wage, the optimal cutoff value, as well as employment for a given firm type in the $X$-sector can be determined regardless of factor endowments and conditions in the $Y$-sector.

![Figure 4: Cutoff Task and Equilibrium Wage in a Mixed Equilibrium](image)

To show how a reduction of border-crossing costs affects the point of intersection of the two curves in Figure 4, we start by observing that lowering $T$ shifts the marginal cost curves (29) and (30) in Figure 3 downward, while it leaves (28) and (21) unaffected. In regimes in which fragmented production or production abroad co-exist with domestic production, the downward shift of the $C_M/ (w\varphi_{Mf})$ and $C_M^*/ (w\varphi_{M^*})$ curves results in higher values of $\tilde{t}_1$. The effect on the equilibrium relative wage can be inferred from Figure 4: the $FE$ line shifts upward, causing a decline in the equilibrium value of $\omega$. The economic intuition behind this effect is straightforward: lowering transport costs reduces the costs of firms engaged in offshoring. To sustain the mixed production regime, foreign production costs relative to domestic ones have to increase. This is brought about by a decline of $\omega$. Hence, as long as domestic and multinational firms co-exist, decreasing
transport costs expand the range of tasks performed domestically by multinational firms.

The effect of a decline in $\Delta$ for a regime in which domestic and multinational firms coexist may also be determined analytically. Condition (26) implies $a_D \omega \phi_k = L_k \omega + L_k^*$ or

$$
\omega = \frac{L_k^*}{a_D \phi_k - L_k}.
$$

(36)

with $k \in \{M^f, M^n\}$. Inserting from (9)–(14) and taking the derivative yields $\partial \omega / \partial T > 0$.

If $M^f$ and $M^n$ firms co-exist, the effect of a decline in $\Delta$ on $\omega$ and $\tilde{\tau}_1$ is not obvious ex ante (both “marginal cost curves” shift downward in Figure 3). To understand how lowering $\Delta$ affects the cutoff value in such a regime, we start by observing that $C_{M^f} / \phi_{M^f} = C_{M^n} / \phi_{M^n}$ requires $\phi_{M^n} [L_{M^f} w + L_{M^f}^* w^*] = \phi_{M^f} [L_{M^n} w + L_{M^n}^* w^*]$. Combining this with (26), (27), and (8), we get

$$
\omega = \frac{L_{M^n}^* - \frac{\phi_{M^n}}{\phi_{M^f}} L_{M^f}^*}{\frac{\phi_{M^n}}{\phi_{M^f}} L_{M^f} - L_{M^n}}
\cdot

\left[ B \left( 1 - 2\tilde{\tau}_1 \right) - \frac{A}{n \pi} \sin(2\pi n \tilde{\tau}_1) + T \right] - \frac{\phi_{M^n}}{\phi_{M^f}} \left[ B \left( 1 - 2n m \tilde{\tau}_1 \right) - \frac{A}{n \pi} \sin(2\pi n \tilde{\tau}_1) + n T \right]
\cdot

\frac{\phi_{M^n}}{\phi_{M^f}} (2n a_M \tilde{\tau}_1 + n T) - (2a_M \tilde{\tau}_1 + T).

$$

(37)

Taking the derivative with respect to $T$ yields

$$
\frac{\partial \omega}{\partial T} = -\left( \frac{n \phi_{M^n}}{\phi_{M^f}} - 1 \right) \left( 1 + \omega \right) \frac{1 + \omega}{2a_M \tilde{\tau}_1 + T},
$$

which is negative. Hence, reducing $T$ shifts the FE-curve in Figure 4 downward, resulting in a rise in $\omega$ and a fall in $\tilde{\tau}_1$. This stands in a sharp contrast to the previous cases in which domestic firms co-existed with offshoring firms.

In order to further explore the influence of transport costs $T$ on firms’ offshoring decisions at the extensive and the intensive margin, and to also analyze the comparative-static properties of regimes with only one firm-type, we consider a special case that relies on two parametric assumptions: first, we impose a Cobb-Douglas production technology in sector $Y$, i.e., $\sigma_Y \rightarrow 1$ and $\alpha_R = \alpha_L = 0.5$, and second, we consider a particular distribution of factor endowments such that the relative factor endowment in the South is just the inverse of that in the North.

**Assumption 1** $Q_Y = L_Y^{0.5} R_Y^{0.5}$, $\bar{L} = \delta$, $\bar{R} = 1 - \delta$, $\bar{L}^* = 1 - \delta$, and $\bar{R}^* = \delta$, $(0 < \delta < 1)$. 19
The following three lemmas describe the effect of a decline in $T$ on relative wages and on offshoring at the intensive margin. Moreover, they show how the total volume of offshoring, which we define as domestic firms’ demand for foreign labor ($\sum_j N_j L_j^*$), reacts to a reduction of border-crossing costs.\textsuperscript{17}

**Lemma 1** In a production regime in which domestic ($D$) and multinational ($j = M^a$ or $j = M^f$) firms co-exist, a decline in transport costs $T$

(i) lowers the wage in the North relative to the wage in the South, $\omega \equiv w/w^*$
(ii) reduces offshoring at the intensive margin (i.e. raises the optimal cutoff $\bar{t}_1$)
(iii) raises the total volume of offshoring $N_j L_j^*$ if Assumption 1 is satisfied
(iv) raises the number of multinational firms $N_j$ if Assumption 1 is satisfied.

The discrepancy between the adjustment at the extensive margin and at the intensive margin has a straightforward interpretation: a lower value of $T$ provides an incentive for Northern firms to take advantage of the possibility to relocate parts of the production process to the South. In the aggregate, these decisions lower the relative wage in the North, and this reduces the volume of offshoring at the firm level. If Assumption 1 is satisfied, the expansion of offshoring at the extensive margin apparently dominates such that the total volume of offshoring increases as transport costs decrease.

**Lemma 2** In a production regime in which the two different types of multinational firms co-exist ($j = M^a$ and $k = M^f$), a decline in transport costs $T$

(i) raises the wage in the North relative to the wage in the South $\omega \equiv w/w^*$
(ii) raises offshoring at the intensive margin (i.e., reduces the optimal cutoff $\bar{t}_1$)
(iii) lowers the total volume of offshoring $N_M^a L_M^* + N_M^f L_M^*$ if Assumption 1 is satisfied.

\textsuperscript{17}For proofs, see Appendix A.
A reduction of $T$ has a stronger effect on the marginal costs of fragmented firms than on firms that choose production abroad. This provides an incentive to move from production abroad to fragmented production and to repatriate tasks that have been offshored before. As a result, the overall volume of offshoring decreases, demand for domestic labor increases, and this raises the relative wage.

**Lemma 3** In a production regime that involves only one multinational firm type ($j = M^a$ or $j = M^f$) and if Assumption 1 is satisfied, a decline in transport costs $T$

(i) raises the relative wage $\omega$, raises offshoring at the intensive margin (lowers the cutoff $\tilde{t}_1$), and lowers the total volume of offshoring $N_j L_j^*$ if $\lambda_j \equiv L_j / L_j^* > 1$

(ii) lowers the relative wage $\omega$, lowers offshoring at the intensive margin, and raises the total volume of offshoring $N_j L_j^*$ if $\lambda_j < 1$

(iii) raises the number of offshoring firms $N_j$.

In a pure production regime, the number of offshoring firms increases as the costs of transportation decrease. This is reminiscent of the result we presented in Lemma 1. However, the effect on wages, firms’ offshoring decision at the extensive margin, and the overall volume of offshoring depends on relative employment in the $X$-industry, i.e., $\lambda_j \equiv L_j / L_j^*$. The reason is that the relative wage $\omega$ depends on $\lambda_j$, and that a change in $T$ directly influences both $L_j$ and $L_j^*$ (see (10) and (11) as well as (13) and (14)). If, initially, $\lambda_j > 1$, the effect of decreasing transport costs on foreign employment in the $X$-sector dominates, and both $\lambda_j$ and $\omega$ increase if $T$ decreases. The higher wage raises offshoring at the intensive margin and lures labor out of the $Y$-sector in the North. In the South, the higher value of $\omega$ results in an expansion of the $Y$-sector. Since this leaves less labor for the $X$-sector, the total volume of offshoring necessarily decreases. The increasing number of offshoring firms and the expansion of tasks delegated to the South are compatible with a lower total volume of offshoring since – with a decreasing $T$ – the South uses less labor to ship intermediate goods across the border. Conversely, if $\lambda_j < 1$ initially, lowering $T$ reduces domestic employment in the $X$-sector by more than foreign employment, such that $\lambda_j$ and $\omega$ decrease. As a result, offshoring at the intensive margin decreases, but
an increasing share of the Southern labor force is employed in the $X$-sector, implying a greater total volume of offshoring. The latter finding is compatible with a higher value of $\tilde{t}_1$ since the number of firms engaged in offshoring increases.

Note that our analysis has characterized the comparative static properties of different production regimes, but has not spelled out the process that leads from one regime to another one. Nevertheless, there are some important qualitative findings to take away: first, decreasing transport costs do not necessarily raise the total volume of offshoring. They do so if multinational firms co-exist with domestic firms (see Lemma 1), but they don’t if some firms choose fragmented production and others choose production abroad. Whether they do so in a pure production regime depends on initial employment in the $X$-sector. Moving from a partial-equilibrium perspective (as in Harms et al. 2012) to general equilibrium adds two important aspects to the offshoring equilibrium. First, within a particular production regime, there is a smooth adjustment to changing transport costs at the intensive and the extensive margin. Second, the volume of offshoring affects relative wages, and this has a repercussion on offshoring at the firm level. In fact, the share of tasks shifted abroad may actually decrease while the total offshoring volume increases.

4 A Numerical Appraisal

In this section, we run numerical simulations to further explore the comparative static properties of our model. In particular, we drop Assumption 1 – i.e., we return to a more general specification of the production function and of factor endowments – and we extend the analysis by also considering the effects of other model parameters. As outlined above, our model allows for two dimensions along which the extent of offshoring changes as exogenous parameters vary: first, the share of the production process that is performed abroad for a given firm-type may increase or decrease (the intensive margin). Second, the number of firms of a certain type may vary (the extensive margin). In what follows, we will analyze how offshoring reacts at the extensive and at the intensive margin to a decline in transport costs, changes in relative productivities and other properties of the production process.
4.1 Calibration

The two regions are scaled so that initially about half of the domestic consumption of good $Y$ is imported from the South, while $X$ goods are produced only by Northern firms and exported to the South. With preference parameter $\beta = 0.5$, the North is endowed with one third of the world’s $R$ and two thirds of the world’s $L$, while the South is endowed with the rest.\footnote{Note that our framework adopts elements from both the Ricardian and the Heckscher-Ohlin framework, i.e., trade is driven by differences in factor endowments and by technological differences. While our assumption that the North is relatively abundant in labor may seem unjustified at first glance, $L$ should be interpreted as effective labor supply, which is determined by both demographic and technological factors.} In industry $X$, we set $\sigma_x = 4$ and $n = 2$ as benchmark values for the substitution elasticity and the number of cycles of $a_M^*(t)$, respectively. We choose somewhat arbitrarily – but within the ranges consistent with our theoretical constraints – the fixed costs and the cost of transporting intermediate goods across borders: $F_D = 1.0; F_{MF} = 1.3; F_{MA} = 1.5; T = 0.2$. With this functional form and parameter values, we calibrate the key technological parameters – $A$, $B$, $a_D$ and $a_M$ –, so that initially about half of the total $X$ output is produced by $D$-type firms and the other half by $M'$-type firms. Appendix B reports the benchmark parameter and equilibrium variable values.

4.2 Factor Endowments and Production Regimes in Equilibrium

Given our benchmark parameter values, Figure 5 presents the equilibrium production regimes for different allocations of production factors between the two regions. The vertical axis is the total world endowment of effective labor, and the horizontal axis is the total world endowment of the composite factor, with the North measured from the southwest (SW) and the South from the northeast (NE).
We see that the equilibrium regimes are associated with differences in relative factor endowments. Intuitively, if the North is highly abundant in effective labor—such that this factor is relatively cheap—no offshoring occurs. Conversely, if the South is highly abundant in labor, we may expect that most firms produce abroad. For intermediate allocations of factors of production, fragmented firms should dominate.\textsuperscript{19}

Figure 6 displays the equilibrium number of each firm-type along the NW-SE diagonal—the two regions differ in relative factor endowments—and along the SW-NE diagonal—the two regions have identical relative factor endowments but differ in size.

\textsuperscript{19}Indeed, altering the distribution of the world endowment in much finer steps, we also have regimes in which only fragmented firms exist between the two regimes of \(\{D, M^I\}\) and \(\{M^I, M^o\}\).
As we move from left to right in panel (a) of Figure 6, the relative share of the North in global labor supply decreases, and its share in the composite factor increases. This raises the Northern wage rate, making offshoring more attractive and thus reducing the number of domestic firms. First, we observe the emergence of fragmented firms that allocate their production process to different regions. Eventually, as the bulk of global labor is located in the South, firms decide to offshore the biggest possible part of the value-added chain, leaving only $M^o$ firms in business.

The effect of increasing the size of the North – leaving relative factor endowments constant – is depicted in panel (b) of Figure 6. As the North is growing bigger, it hosts an increasing share of the global labor supply. Since the labor endowment is decisive for the attractiveness of offshoring, the picture is first dominated by multinationals producing in the South, and eventually by domestic firms. At intermediate stages – i.e., for constellations at which both regions are of roughly equal size – most of global production is performed by fragmenting multinationals that intensively exploit international cost differences.

### 4.3 The Effects of Globalization

We now investigate the effects of globalization, which we interpret as a decline in $T$. Figure 7 reports the effects of a decline in $T$ (horizontal axis) on the prevalence of each firm-type. Note that the horizontal axis is inverted, moving from higher to lower values – i.e., the extent of globalization increases (with transport costs decreasing) from left to right.
Figure 7 supports the theoretical results derived above: if $T$ decreases, the number of multinational firms increases. As long as transportation costs are high, production abroad dominates the picture. With lower values of $T$, fragmented production replaces production abroad and, eventually, this becomes the dominant mode of production.

Figure 8 presents the induced variations in cutoff task $\tilde{t}_1$ and the relative wage $\omega$ (with initial values normalized to one at $T = 0.25$).
As already shown in the comparative static analysis, a decline in transport costs first raises the cutoff value $\tilde{t}_1$ and thus lowers the range of tasks offshored. This is due to the fact that a higher demand for labor in the South, resulting from an increasing number of offshoring firms, lowers the wage ratio $(w/w^*)$, as also shown by Figure 8. According to equation (21), this makes it less attractive to offshore a large range of tasks. However, for very low levels of $T$, as only fragmented firms exist, the effect of a decline in $T$ on the cutoff values is reversed. This reflects case (i) spelled out in Lemma 3: apparently, lowering transport costs and thus releasing labor in both regions, has a bigger impact on the South. As shown by Figure 8, this raises $w/w^*$ so that it is profitable to offshore a higher range of tasks to the South.

### 4.4 Technological Change

In this subsection, we explore how technological changes affect the relative importance of alternative production modes. Again, the horizontal axis is inverted, moving from higher to lower values – i.e., productivity increases are reflected by decreasing input coefficients from left to right.

Figure 9 shows the effects of domestic technological change of $a_D$ and $a_M$. Intuitively, technological progress in domestic firms – a fall in $a_D$ – makes these firms more competitive compared to multinationals. More entry by domestic firms raises the domestic wage, which is more detrimental to fragmented firms given that they perform more tasks in the North than firms producing almost everything abroad. In contrast, productivity improvements of multinational firms – a fall in $a_M$ – reduces the number of domestic firms. Between the two types of multinational firms, fragmented firms benefit more by the same logic as above.
Figure 9: Technological Change of $a_D$ and $a_M$

Figure 10 focuses on the South and presents the effects of varying the parameters $A$ and $B$. Recall that raising $A$ increases the amplitude of the $a_M^*(t)$ function, representing greater cost differences between different tasks in the production chain. Conversely, reducing $A$ implies a decline in the cost advantage of producing in the South for tasks $t \in [\tilde{t}_1, \tilde{t}_2]$ and $[\tilde{t}_3, \tilde{t}_4]$. This makes fragmentation less attractive, and eventually only domestic firms and production abroad firms prevail. A similar pattern emerges if $B$ – i.e., the average costs associated with producing abroad – increases: in this case, the number of firms choosing any type of offshoring declines.

Figure 10: Technological Change of $A$ and $B$

Finally, Figure 11 displays the effect of an increase in the number of cycles ($n$) on the relative importance of alternative production modes. Recall that we interpret production processes that are characterized by a higher value of $n$ as being more “sophisticated”, reflecting a more complex structure of cost differences along the value-added chain.
Increasing $n$ has similar effects as an increase in $T$. Given that fragmented offshoring requires transportation costs of $n$-times $T$ in each region, raising $n$ reduces $N_{M^d}$ while the number of domestic firms increases.

5 Summary and Conclusions

In this paper, we have analyzed the extent of offshoring in a two-region general equilibrium model that is based on three crucial assumptions. First, firms’ production process follows a rigid structure that defines the sequence of production steps. Second, domestic and foreign relative productivities vary in a non-monotonic fashion along the production chain. Third, each task requires the presence of an unfinished intermediate good whose transportation across borders is costly. We believe that these assumptions are quite plausible for a wide range of industries. As a consequence, some firms may be reluctant to offshore individual production steps, even if performing them abroad would be associated with cost advantages: the reason is that adjacent tasks may be cheaper to perform in the domestic economy and that high transport costs do not justify shifting the unfinished good abroad and back home.
Using this basic structure and setting up a simple general equilibrium model along these lines, we have analyzed the influence of globalization – interpreted as a variation in border crossing costs – and of technological changes on the volume of offshoring at the extensive and the intensive margin. As relative production costs vary, firms adjust the share of tasks they perform abroad (the intensive margin). At the same time, the number of firms that fragment their production process or produce almost entirely abroad varies (the extensive margin). Both adjustments may affect relative wages at home and abroad, which can reinforce or dampen the initial impulse. We have shown that globalization in the form of declining transport costs may have different effects on offshoring at the extensive and intensive margin. For example, if domestic and multinational firms coexist, lower transport costs result in a decrease of offshoring at the intensive margin – i.e., firms offshore a smaller part of the entire production process – but an increase in the number of firms that perform at least some tasks abroad.

We believe that the simplicity of our model – in particular, the symmetry of the $a^*_M$ function – has allowed us to derive some novel results, which are likely to carry over into a more general environment. The challenge ahead is to expand the framework to accommodate additional features of reality, e.g., by introducing a non-symmetric shape of the $a^*_M(t)$ function, or by allowing for transport costs that vary along the production chain. The second challenge is to gauge the relative importance of sequential production processes for the economy as a whole. Our contribution rested on the assumption that all firms have to cope with a rigid sequence of production steps. This may be as unrealistic as the notion that production processes can be re-arranged freely by every firm. We believe that characterizing real-world production processes in terms of “sequentiality” holds ample promise for future research.

References


Appendix A: Proof of the Lemmas

Proof of Lemma 1: Parts (i) and (ii) follow from equation (36). For parts (iii) and (iv), first note that the zero profit condition (5) and the factor demand condition (4) together with the assumed Cobb-Douglas technology imply 

\[ 2(\omega \rho) = 1 = 2(\omega \phi) = 2(\rho \phi) \]

for North and South. Equilibrium on the market for good \( Y \) requires

\[ Q_Y + Q_Y^* = (1 - \beta) \left[ w\bar{L} + r\bar{R} + w^*\bar{L}^* + r^*\bar{R}^* \right] \quad \text{or} \]

\[ 2[r(1 - \delta) + r^*\delta] = (1 - \beta) [w\delta + w^*(1 - \delta) + r(1 - \delta) + r^*\delta] \]

With \( wr = w^*r^* \), which follows from the free tradability of good \( Y \), this yields

\[ \frac{r^*}{w^*} = \frac{\omega^2}{1 + \beta} \omega \quad \text{and} \]

\[ L_Y^* = \frac{\omega^2 \delta}{1 - \delta} L_Y = \left( \frac{1 - \beta}{1 + \beta} \right) \omega \delta \]  
(A.1)

A decline in \( \omega \), induced by a reduction of \( T \), lowers \( r^*/w^* \) and raises \( r/w \). \( L_Y \) increases and \( L_Y^* \) declines. Since \( \sigma_x N_j L_j^* F_j = 1 - \delta - L_Y \), a decline in \( L_Y^* \) raises \( N_j L_j^* \) and (because \( L_j^* \) declines) also \( N_j \).

Proof of Lemma 2: Parts (i) and (ii) follow from equation (37). For part (iii), we can apply (A.1) and the fact that the volume of offshoring equals \( 1 - \delta - L_Y^* \).

Proof of Lemma 3: Inserting for \( L_j \) and \( L_j^* \) from (A.1) into \( \sigma_x N_j L_j^* F_j = 1 - \delta - L_Y \) and \( \sigma_x N_j L_j F_j = 1 - \delta - L_Y^* \) determines \( \lambda_j \) as

\[ \lambda_j^f(\omega) = \frac{\delta - L_Y}{1 - \delta - L_Y^*} = \frac{\omega \delta (1 + \beta) - (1 - \delta) (1 - \beta)}{(1 - \delta)(1 + \beta) - \omega \delta (1 - \beta)}. \]  
(A.2)

According to (A.2), \( \lambda_j^f(\omega) \) increases in \( \omega \), i.e., \( \partial \lambda_j^f / \partial \omega > 0 \). The second equation that determines the equilibrium can be derived by dividing the labor demand equations (10) and (13) for \( j = M^f \) or (11) and (14) for \( j = M^a \) and inserting \( t_1^f = t_1(\omega) \). The resulting ratio can be written as \( \lambda_j^H(\omega, T) \), with \( \partial \lambda_j^H / \partial \omega < 0 \). For the partial derivative \( \partial \lambda_j^H / \partial T > 0 \), we obtain

\[ \frac{\partial \lambda_j^H}{\partial T} = \frac{n L_j^* - n L_j}{(L_j^*)^2} < 0 \quad \text{if} \quad \lambda_{M^f} > 1 \quad \text{and} \]

\[ \frac{\partial \lambda_j^H}{\partial T} = \frac{L_j^* - L_j}{(L_j^*)^2} < 0 \quad \text{if} \quad \lambda_{M^a} > 1. \]
Setting $\lambda_j'(\omega) = \lambda_j''(\omega, T)$ yields the equilibrium $\omega$, and from taking the derivative, we can determine the following condition:

$$
\frac{d\omega}{dT} = \frac{\partial \lambda_j'' / \partial T}{\partial \lambda_j'/\partial \omega - \partial \lambda_j'' / \partial \omega}.
$$

Equation (A.3) implies $d\omega/dT < 0$ if $\lambda_j > 1$. In this case, a decline in $T$ raises $\omega$ and lowers $\tilde{t}_1$. This completes the proof for part (i). Part (ii) can be proven analogously. For part (iii) in the case of $\lambda_j > 1$, we first note that $L_j$ decreases if both $T$ and $\tilde{t}_1$ decline – according to (10) or (11). Second, $L_Y$ declines according to (A.1), which implies that $N_jL_j$ increases. Since $L_j$ declines, this is only possible if $N_j$ increases. In the case of $\lambda_j < 1$, a decline in $T$ results in an increase in $\tilde{t}_1$ such that $L_j^*$ declines – according to (13) or (14). As $L_Y^*$ declines according to (A.1), $L_j^*N_j$ has to increase, which, again, implies that $N_j$ increases.
Appendix B: Benchmark Parameter and Variable Values

<table>
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<th>$\bar{R}$</th>
<th>$\bar{L}$</th>
<th>$\bar{R}'$</th>
<th>$\bar{L}'$</th>
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<th>$\sigma_\tau$</th>
<th>$\sigma_\alpha$</th>
<th>$\alpha_\gamma$</th>
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<th>$T$</th>
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<th>$F_{M^s}$</th>
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<th>$Y$</th>
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<th>$x^2_{M^f}$</th>
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